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Abstract. We investigate the problem of companies that want to cooperate either by combining their salesforces or by operating a joint salesforce. Companies may have salesforces of different sizes that also differ in their effectiveness. They need an instrument to evaluate how much they gain from a cooperation, and a mechanism to allocate the profit in a fair way. In the typical case of a lack of response data we suggest to infer additional sales based on response functions for which the Dorfman-Steiner theorem is holding in the optimum. Searching for an appropriate allocation mechanism becomes difficult because typical cooperative solutions where each company pays its own salesforce but benefits from increased sales, or where commission rates on sales are paid to a joint subsidiary, may lead to asymmetric distributions of profit contribution across companies. We suggest that companies follow the Nash-solution for cooperative games which recommends that each company receive in advance the profit it would achieve in the case of non-cooperation, and that the remaining profit be shared equally.

Zusammenfassung. Unternehmen können im Bereich des Verkaufs kooperieren, indem sie ihre Außendienste kooperativ einsetzen oder ganz zusammenlegen. Wenn Außendienste in ihrer Größe und Effektivität unterschiedlich sind, benötigen Unternehmen für ihre Verhandlungen eine Methode, mit der sie die zusätzlichen Umsätze prognostizieren können, sowie einen Mechanismus, nach dem die Außendienst-Kosten aufgeteilt werden sollen. Da in der Regel Daten für die Abschätzung der Umsatzreaktion fehlen, wird hier vorgeschlagen anzunehmen, daß Unternehmen im Optimum operieren, so daß die gesuchten Elastizitäten aus dem Dorfman-Steiner-Theorem abgeleitet werden können. Die Allokation der Außendienstkosten ist nicht trivial, weil typische Praxis-Lösungen zu unfairen Verteilungen führen können. Wir empfehlen deshalb, nach der Nash-Lösung für kooperative Spiele vorzugehen und den ge-

meinsamen Deckungsbeitrag so aufzuteilen, daß zunächst jedes Unternehmen den gegenwärtigen Deckungsbeitrag (als Drohpunkt) erhält, um dann den Rest hälftig zu teilen.

Key words: Salesforce management, cooperation, cost allocation, cooperative Nash-solution

Schlüsselwörter: Verkaufsaußendienst, Kooperation, Kosten-Allokation, Kooperative Nash-Lösung

1 Problem

The use of a field salesforce becomes more and more expensive. The costs of a customer call have risen from 100 US$ at the end of the 1970s to 500 US$ in 1995 [6]. Expanding a salesforce usually generates additional sales only at a decreasing marginal rate. This situation has often led to the decision to decrease rather than increase salesforce size. However, there exists an alternative for companies that intend to expand. More and more companies look for cooperations with field salesforces of other companies carrying non-competing products but selling to the same customers. Cooperating salesforces or even joint salesforces can be profitable if the savings in salesforce calling- and travel-time from avoiding to call on the same account twice is higher than the additional calling-time needed for presenting a larger assortment. The companies may initially operate with salesforces of different sizes that may differ in their effectiveness. In this situation, it is not clear beforehand whether companies benefit from a cooperation. Thus, companies need instruments for evaluating how much they gain from a cooperation. Surprisingly, this problem has – to the best of our knowledge – never been treated in the literature although cooperation of salesforces is a topic of increasing importance in practice.

Instruments for an evaluation of the additional profit from such a cooperation have to address two problems. First, the company has to predict the additional sales from a cooperation. Second, joint costs of the salesforces have to be allocated across companies. With appropriate methods, companies can assess whether and under which conditions a cooperation is profitable. Prediction of additional sales is a challenging task if companies only have data on their sales and costs but not on sales response. This is usually the case and even more of a problem for data on the prospective partners. Therefore, we develop a method that allows us, under certain assumptions, to calibrate the required sales response functions on the basis of profit and cost data only. However, even when sales resulting from cooperation can be predicted, calculation of profit remains to be equivocal because it may not be possible to separate the salesforce costs for the products of cooperating companies. Based on the proposed calibration of sales response functions we show that typical cooperative solutions where each company pays its own salesforce but benefits from increased sales, or where commission rates on sales are paid to a joint subsidiary may lead to a very asymmetric distribution of profit contribution across companies. Consequently, we have to look for allocations that are considered to be fair by all cooperating companies so that an agreement is reached. We suggest an allocation based on the results of the theory of cooperative games. In particular, we recommend that companies follow the cooperative Nash-solution, which is the only solution that meets certain fairness criteria. It postulates that in a fair cooperation each company receive in advance the profit it would achieve in the case of non-cooperation (threat point), and that the remaining profit be shared equally.

In order to illustrate the managerial relevance of our problem we discuss it on the basis of a case study. While the data reported in this paper are structurally valid they are transformed in order to protect the firm that provided the case. In section 2 we describe the case of two companies that are in the process of negotiating a cooperation. Based on the Dorfman-Steiner theorem and the assumption that companies are operating near to the optimum we derive values for the elasticities of sales with respect to changes of salesforce and advertising budgets. In section 3 we propose specific sales response functions that can be calibrated with these elasticities. This allows for an evaluation of the additional profit contribution of a cooperation in form of combining salesforces. In section 4 we compare this to the solution holding for a joint salesforce. Since in our case the profits are distributed very asymmetrically, we investigate whether alternative allocations of salesforce costs across companies lead to fairer solutions. Costs could be allocated either proportional to profit contribution and the respective elasticities or only proportional to profit contribution (implemented through commissions on the respective sales volumes). We show that these solutions are still not very satisfactory. Therefore, we propose in section 5 to follow the Nash-solution for cooperative games. We evaluate it with respect to additional profit contribution, and discuss how to implement such a solution in practice. The paper closes with a discussion of limitations and managerial implications.

2 Current situation

In order to illustrate the managerial relevance and to facilitate the presentation, we discuss our problem based on data of a case study with two companies. Of course, the derived conclusions and solutions can be generalized to any number of cooperating companies. Imagine a company A that produces and sells consumer products through a field salesforce to retailers and restaurants. This company looks for opportunities to increase its profit from additional sales. Company A feels that it is already close to an optimum with respect to its salesforce size and advertising budget level. Therefore, it wants to investigate whether it can increase profits by cooperating with another company. Prerequisite of such a cooperation is that the products do not compete against each other and that the companies serve the same customers. Similar considerations are now undertaken at headquarters of some pharmaceutical companies (e.g. Pfizer Inc., USA) where it is already usual that their salesforces also carry products of other pharmaceutical companies. At a first glance, this problem appears to be similar to the one where a company wants to assess whether it is more profitable to operate with one general salesforce or with two or more specialized salesforces. This is called the salesforce structuring problem [14] and compares the advantages of product specialization with the advantages of avoiding multiple calls [13]. In contrast to that, the companies in our problem do not have sufficiently detailed data on the operations of the potentially cooperating partner making it infeasible to assess the profitability of the cooperation by just solving a complete salesforce structuring problem. Furthermore, cooperation deals with independent companies that involves the additional problem of how to allocate the surplus across companies in a fair way.

In our case, after an intensive search, company A has found company B which offers complementary products. During the negotiations about the cooperation both companies exchanged the data in Table 1 which describe the current situation.

Current Situation	Company A	Company B	Total
Sales Volume	120,000	60,000	180,000
Production Costs	54,000	33,000	87,000
Profit Contribution I	66,000	27,000	93,000
Salesforce Budget	13,200	4,320	17,520
Advertising Budget	6,600	2,700	9,300
Profit Contribution II	46,200	19,980	66,180

Table 1: Current sales, marketing budgets, and profit contribution in thousand DM (TDM)

First, company A wants to know what the effect of the cooperation on its sales would be. As a cooperation offers a better utilization of salesforce time and may require an adjustment of the advertising budget we need information on the elasticities of sales with respect to changes in the salesforce budget and the advertising budget. In general, such elasticities can be estimated reliably with the help of market research data. However, results from market research are often not available in practice and cannot be produced on short notice. This was also true for our company under investigation. It started cooperation talks and wanted to come to an agreement within a rather short period of a few months. No response data were available. The only data that could be used was the one in Table 1. Because of the time horizon it was not possible to carry out a study the results of which allow for a calibration of the required response functions. Furthermore, it was impossible for our company under investigation to get such response data from the potentially cooperating partner. If we, nevertheless, want to support the company's assessment of the benefits of a cooperation, we have to search for methods that can infer the required elasticities from the readily available but very limited accounting data (sales and costs) as given in Table 1.

Such a method is suggested here. It is based on the assumption that the companies have already optimized their salesforce and advertising budget levels. At a first glance, this assumption appears to be restrictive, but given that both companies have been operating successfully in their markets for several decades, it is plausible that they are close to an optimum. Similar arguments are used by Darwinian economics [4]. In addition, company A subjectively judged the budgets of companies A and B to be close to an optimum. Based on this assumption we can use the theorem by Dorfman and Steiner [7] which states that in the optimum the ratio of the salesforce budget divided by profit contribution I (before marketing cost) equals the salesforce elasticity. The same is true analogously for the advertising budget and elasticity. Note that we make this assumption of optimality only because it allows us to derive elasticity values for which we had no other information. In more detail, we can derive the optimality condition for the salesforce budget as follows:

$$\pi = g \cdot S(V,W) - V - W \quad \Rightarrow \max! \tag{1}$$

π : profit contribution II (after marketing costs),
g : gross margin,
S : sales volume,
V : salesforce budget,
W : advertising budget.

$$\frac{\partial \pi}{\partial V} = g \cdot \frac{\partial S}{\partial V} - 1 \overset{!}{=} 0 \tag{2}$$

$$g \cdot \underbrace{\frac{\partial S}{\partial V} \cdot \frac{V}{S}}_{\text{Elasticity } \beta} \cdot \frac{S}{V} = 1 \tag{3}$$

$$g \cdot \beta \cdot S = V \tag{4}$$

$$\beta = \frac{V}{g \cdot S} = \frac{\text{Salesforce Budget}}{\text{Profit Contribution I}} \tag{5}$$

Analogously, the same holds for the advertising budget:

$$\gamma = \frac{W}{g \cdot S} = \frac{\text{Advertising Budget}}{\text{Profit Contribution I}} \tag{6}$$

with γ as elasticity of sales with respect to changes of the advertising budget.

In order to infer the elasticities that hold if the current solution really is an optimum, we have computed the gross margins and the respective ratios of the salesforce budget and advertising budget to sales volume as well as to profit contribution I (before marketing costs). These percentages are displayed in Table 2.

Ratios	Company A	Company B	Total
Gross Margin	55.00%	45.00%	51.67%
Salesforce Budget as % of Sales Volume	11.00%	7.20%	9.73%
Salesforce Budget as % of Profit Contribution I	20.00%	16.00%	18.84%
Advertising Budget as % of Sales Volume	5.50%	4.50%	5.17%
Advertising Budget as % of Profit Contribution I	10.00%	10.00%	10.00%

Table 2: Gross margins and budget ratios in the current situation

From Table 2 we learn that company A, by assuming optimal budgets, also assumes according to equations (5) and (6) that its elasticities with respect to changes of the salesforce budget and advertising budget are 0.2 and 0.1, respectively. If company A applies the same assumption to company B, then the budgets of company B imply salesforce and advertising elasticities of 0.16 and 0.1, respectively. These values are lying in the range of reported elasticity values (see [5], [8], [9], and [15]) and are thus quite plausible. Of course, this is not a proof that the values are correct. However, compared to no information at all, these infered values can give valuable insights into the evaluation task. It is also interesting to note that by assuming optimality of current operations we can apply a very simple method to infer elasticities of marketing instruments.

3 Profitability of cooperating salesforces

If companies A and B want to keep their salesforces, but agree to cooperate, they can divide their joint accounts into two groups and assign them exclusively to the two salesforces. This avoids double calls on accounts. As the salesforces of both companies carry the complete assortment of both companies, company A gains access to additional calling-time proportional to the size of the salesforce of company B. At the same time, the salespersons have to present the assortments of both companies, which reduces the effective calling-time available at each call for the products of each company. Assume that the salesforce budget is spent on the remuneration of salespersons and on costs for calling on customers. Then, the salesforce calling-time is proportional to the budget. Thus, we consider budgets and time to be interchangeable. Let the effective calling-time as a percentage of total calling-time be denoted by m. Then, the effective calling-time and thereby the effective salesforce budget for the i-th company can simply be operationalized by the percentage m_i of the sum of the salesforce budgets $V_A + V_B$.

Note that the sum of m_A and m_B is not necessarily equal to 1. Rather, the calling-time required for presenting both assortments together at one visit is generally less than the sum of the calling-times in the case that both assortments are presented separately. This is due to set-up times per call and synergies in the presentation. If the required time for the joint presentation of both assortments is for example 80% of the calling-times for both calls, then m_A and m_B should sum up to 1.6. In our case, company A assumes that the same parameter value m = 0.8 holds for both companies. However, similar results would hold in the following if we assumed different values for m across companies. Now, if the salesforce elasticities β_i of the companies $i \in$ A,B are known (as derived from our optimality conditions (5)) it is possible to estimate the expected sales volume S_i^{coop} in the case of a cooperation according to the following equation:

$$S_i^{coop} = S_i^{current} \cdot \left(\frac{m \cdot \sum_{j \in A,B} V_j}{V_i} \right)^{\beta_i} \qquad (i \in A,B) \tag{7}$$

It is emphasized that we only consider sales response at the aggregate level of a salesforce because other more detailed data at the customer level was not available. In equation (7) we multiply the current sales volume $S_i^{current}$ with the ratio of calling-time in the case of cooperation to the current calling-time risen to the power of the elasticity β_i. This type of response function has been proposed in [2] and [3]. It is equivalent to the multiplicative function with a scaling constant of $S_i^{current}/(V_i^{\beta_i})$. Note that function (7) implies a constant elasticity which may not be the case in practice. However, the multiplicative response function is the most commonly used response function in marketing [8, p.38]. With this approach for predicting the sales volumes we arrive at the data for cooperating salesforces presented in Table 3.

Cooperation	Company A	Company B	Total
Sales Volume	121,448	72,433	193,881
Production Costs	54,652	39,838	94,490
Profit Contribution I	66,796	32,595	99,391
Salesforce Budget	13,200	4,320	17,520
Advertising Budget	6,600	2,700	9,300
Profit Contribution II	46,996	25,575	72,571
Current Situation	46,200	19,980	66,180
Additional Profit	796	5,595	6,391

Table 3: Sales, marketing budgets, and profit contribution in thousand DM (TDM) for cooperating salesforces

Compared to the current situation, company A achieves a slightly higher sales volume and profit contribution. However, company B gains much more from the cooperation. It can increase its sales volume by 20.7% and its profit contribution II (after marketing expenses) by 28.0%. This implies that the smaller company B takes advantage of the larger salesforce of company A utilizing it for its own purposes. On the other hand, company A can only expect a small additional profit, even though it has a higher elasticity for its salesforce and a higher gross margin.

This creates a very asymmetric profit contribution situation which is not very attractive for company A that sacrifices its large salesforce, but gains only a small additional profit. In such a situation it is understandable that company A has an interest in searching for other agreements that offer a fair solution to both companies.

4 Profitability of a joint salesforce

An alternative to a cooperation of legally separate salesforces would be to set up a joint salesforce and to run it through a joint subsidiary. This subsidiary determines a salesforce budget that is optimal from a joint point of view. We assume for the moment that the advertising budgets remain constant at the current levels. More specifically, the subsidiary maximizes equation (8) with respect to the two salesforce budgets V_A and V_B:

$$\pi = g_A \cdot S_A(m \cdot \sum_{j \in A,B} V_j, W_A) - V_A - W_A$$
$$+ g_B \cdot S_B(m \cdot \sum_{j \in A,B} V_j, W_B) - V_B - W_B \Rightarrow \max \quad (8)$$

Let $\tilde{V} = \sum_{j \in A,B} V_j$. Note that m does not differ across companies because the total time is affected in the same way for both companies. Then the first derivative of π with respect to $\tilde{V}$ is given by equation (9). Setting this equation equal to zero, results in equation (11) which describes the optimal joint salesforce budget.

$$\frac{\partial \pi}{\partial (m \cdot \tilde{V})} = g_A \cdot \frac{\partial S_A}{\partial (m \cdot \tilde{V})} \cdot m + g_B \cdot \frac{\partial S_B}{\partial (m \cdot \tilde{V})} \cdot m - 1 \overset{!}{=} 0 \quad (9)$$

$$g_A \cdot \underbrace{\frac{\partial S_A}{\partial (m \cdot \tilde{V})} \cdot \frac{m \cdot \tilde{V}}{S_A}}_{\beta_A} \cdot \frac{S_A}{m \cdot \tilde{V}} \cdot m + g_B \cdot \underbrace{\frac{\partial S_B}{\partial (m \cdot \tilde{V})} \cdot \frac{m \cdot \tilde{V}}{S_B}}_{\beta_B} \cdot \frac{S_B}{m \cdot \tilde{V}} \cdot m = 1 \quad (10)$$

$$g_A \cdot \beta_A \cdot S_A + g_B \cdot \beta_B \cdot S_B = \tilde{V} \quad (11)$$

As can be seen from equation (11), the joint salesforce budget depends on the gross margins, the salesforce elasticities, and the current sales volumes of both companies. However, equation (11) does not indicate which parts of the salesforce costs have to be borne by the companies. We, therefore, want to investigate which mechanism for allocating the salesforce costs or resulting profits leads to a fair solution that will be accepted by both companies.

In this section, we first want to investigate two common sense allocation mechanisms, one that may be derived naively from equation (11), and another one that company A considers to be plausible. These serve as benchmarks to a theoretically derived solution that is discussed next in section 5.

A closer look on equation (11) suggests that the optimal joint salesforce budget consists of two additive parts that are given by the (mathematical) products of the gross margin, the salesforce elasticity, and the current sales volume of each company. Therefore, it seems plausible to allocate the salesforce costs proportional to these (mathematical) products:

$$V_i = \frac{g_i \cdot \beta_i \cdot S_i}{\sum_{j \in A,B} g_j \cdot \beta_j \cdot S_j} \cdot \tilde{V} \qquad (i \in A, B) \tag{12}$$

With the advertising budget still fixed at the current level, one arrives at the data in Table 4 for a joint salesforce.

Joint Salesforce	Company A	Company B	Total
Sales Volume	123,211	73,272	196,483
Production Costs	55,445	40,300	95,745
Profit Contribution I	67,766	32,972	100,738
Salesforce Budget	**13,553**	**5,276**	**18,829**
Advertising Budget	6,600	2,700	9,300
Profit Contribution II	**47,613**	**24,996**	**72,609**
Current Situation	46,200	19,980	66,180
Additional Profit	**1,413**	**5,016**	**6,429**

Table 4: Sales, marketing budgets, and profit contribution in thousand DM (TDM) for a joint salesforce

As can be seen from Table 4, a joint salesforce with the allocation of costs according to (12) is more profitable in total than just a cooperation. However, the additional profits of company A and company B are still strongly asymmetric. The total additional profit has increased only marginally compared to the situation of cooperating salesforces. In contrast, the share of profit contribution II (after marketing costs) going to company A has increased from 12.5% to 22.0%. Nevertheless, there is still doubt as to whether company A would agree to such an allocation of costs and profit.

Let us, therefore, investigate the proposal by company A to negotiate a commission based allocation of costs. If the two companies only want to cover the costs of the joint salesforce by the subsidiary while paying for advertising on their own, the subsidiary could request a commission rate on sales volume from the companies. Given that commission rates proportional to gross margins lead to a more profitable allocation of calling-time of the joint salesforce across products than commission rates proportional to sales [1], the level of the commission rates can be determined as follows:

$$c_i = g_i \cdot \frac{\tilde{V}}{\sum_{j \in A,B} g_j \cdot S_j} \qquad (i \in A, B) \qquad (13)$$

c_i: Commission rate on sales of the i-th company.

In our case study this leads to commission rates of 10.28 % on sales of company A and 8.41 % on sales of company B. In the case of sales volumes that are optimal for a joint salesforce we arrive at the solution shown in Table 5. As can be seen there, the sales volumes and the total additional profit remain the same. The only numbers that change are the allocation of salesforce costs and thus the resulting profits across the two companies. Under the commission based allocation rule, company A has to pay less than under the previous rule, thereby achieving a higher profit contribution I (before marketing costs). In consequence, it achieves a share of the additional profit of 35.8% which is more attractive than in all previous solutions. However, this is still an asymmetric allocation of profits. Therefore, we want to search for an allocation of costs that can be derived from the theory of cooperation.

Commission Based Allocation	Company A	Company B	Total
Sales Volume	123,211	73,272	196,483
Production Costs	55,445	40,300	95,745
Profit Contribution I	67,766	32,972	100,738
Salesforce Budget	**12,666**	**6,163**	**18,829**
Advertising Budget	6,600	2,700	9,300
Profit Contribution II	**48,500**	**24,109**	**72,609**
Current Situation	46,200	19,980	66,180
Additional Profit	**2,300**	**4,129**	**6,429**

Table 5: Sales, marketing budgets, and profit contribution in thousand DM (TDM) for a joint sales force with a commission based allocation of costs

5 Profitability of solution derived from the theory on cooperative two-person-games

Instead of the ad hoc solutions discussed so far one may consider allocation mechanisms derived from the theory of cooperation as developed in game theory. Our case is equivalent to a cooperative two-person-game. The theory of cooperative games is concerned with finding a referee solution that will be accepted by all cooperating players. Nash [11,12] has postulated that a solution is considered to be fair if it exhibits the following characteristics:

1. Invariance with respect to linear transformations of the utility function.
2. Individual rationality: The profits of the referee solution for each player must be higher than in the non-cooperative solution.
3. Pareto optimality: The referee solution must be undominated.
4. Symmetry: If the roles of the two players are completely symmetric, then the solution is also symmetric.
5. Independence of irrelevant alternatives.

Nash [12] proves that the optimal solution of the following mathematical program for two players A and B meets all the conditions postulated above:

$$[\text{profit}_A(\text{Coop}) - \text{profit}_A(\text{Non-coop})] \cdot [\text{profit}_B(\text{Coop}) - \text{profit}_B(\text{Non-coop})] \Rightarrow \max! \quad (14)$$

$$\text{Profit}_A(\text{Coop}) \geq \text{profit}_A(\text{Non-coop}) \quad (15)$$

$$\text{Profit}_B(\text{Coop}) \geq \text{profit}_B(\text{Non-coop}) \quad (16)$$

$\text{Profit}_i(\text{Coop})$: Profit of the i-th company from the optimal solution of cooperation,
$\text{Profit}_i(\text{Non-coop})$: Profit of the i-th company from the optimal solution of non-cooperation (threat point).

This means that the players should only share the additional profit that results from the cooperation while receiving in advance that part of the profit which they could have achieved anyway in the case of non-cooperative behavior. The rationale behind this mathematical program is that the profit cannot be shared in total because the players have different threat points, i.e. profits in the case of non-cooperation. It is thereby plausible to share only the additional profit resulting from the cooperation. Constraints (15) and (16) ensure that the players only accept solutions that are better than the one they could achieve in the case of non-cooperation. For our problem this means that we have to search for a solution that maximizes the additional joint profit from cooperation (i.e. by operating a joint salesforce) over the respective profits in the case of non-cooperation (i.e. by operating separate salesforces). This can be achieved with the help of the following mathematical program:

$$\left[g_A \cdot S_A(m \cdot \sum_{j\in A,B} V_j, W_A) - V_A - W_A - \pi_{A,NC}\right] \cdot \left[g_B \cdot S_B(m \cdot \sum_{j\in A,B} V_j, W_B) - V_B - W_B - \pi_{B,NC}\right] \Rightarrow \max! \quad (17)$$

$$g_A \cdot S_A(m \cdot \sum_{j\in A,B} V_j, W_A) - V_A - W_A \geq \pi_{A,NC} \quad (18)$$

$$g_B \cdot S_B(m \cdot \sum_{j\in A,B} V_j, W_B) - V_B - W_B \geq \pi_{B,NC} \quad (19)$$

$$V_A, V_B, W_A, W_B \geq 0 \quad (20)$$

$\pi_{i,NC}$: Profit of the i-th company from the optimal solution of separate salesforces (non-cooperative solution, threat point).

The objective (17) is to maximize the (mathematical) product of the differences between the profit of the cooperative Nash solution for a joint salesforce and the profit of the optimal solution in the case of separate salesforces for both companies. This is achieved if the additional profits beyond the sum of the profits generated by separate salesforces are shared half and half. Constraints (18) and (19) guarantee individual rationality, i.e. that the profits of the referee solution for each company are greater than the profits in the case of separate salesforces. Constraints (20) ensure nonnegative values for the budgets.

The optimal solution for this cooperative Nash solution with advertising budgets still fixed at their current levels, is described in Table 6.

Nash Solution of a Joint Salesforce with fixed Advertising Budgets	Company A	Company B	Total
Sales Volume	123,211	73,272	196,483
Production Costs	55,445	40,300	95,745
Profit Contribution I	67,766	32,972	100,738
Salesforce Budget	**11,751**	**7,078**	**18,829**
Advertising Budget	6,600	2,700	9,300
Profit Contribution II	**49,415**	**23,195**	**72,610**
Current Situation	46,200	19,980	66,180
Additional Profit	**3,215**	**3,215**	**6,430**

Table 6: Sales, marketing budgets, and profit contribution in thousand DM (TDM)

for a cooperative Nash solution with fixed advertising budgets

As can be seen from Table 6, the total additional profit is the same as in the solution of a commission based allocation of costs. This is also true for the sales volumes and profit contributions I. The only difference is that now the costs of the salesforce budget are allocated in a different way, namely by sharing the total additional profit equally between both companies.

If the subsidiary also optimizes the advertising budgets one arrives at the solution given in Table 7.

Nash Solution of Joint Salesforce with optimal Advertising Budgets	Company A	Company B	Total
Sales Volume	123,864	75,059	198,923
Production Costs	55,739	41,283	97,022
Profit Contribution I	68,125	33,776	101,901
Salesforce Budget	11,862	7,168	19,030
Advertising Budget	6,812	3,377	10,189
Profit Contribution II	49,451	23,231	72,682
Current Situation	46,200	19,980	66,180
Additional Profit	3,251	3,251	6,502

Table 7: Sales, marketing budgets, and profit contribution in thousand DM (TDM)
for a cooperative Nash solution with optimized advertising budgets.

Compared to Table 6, both companies achieve slightly higher sales volumes and additional profit by adjusting the advertising budgets to the higher levels chosen for the salesforce budgets.

This structure of the optimal cooperative Nash solution suggests the following procedure for the optimal profit allocation for a joint salesforce: The companies should receive in advance a profit that is equal to their profit contribution II (after marketing costs) of the current situation. This profit can be determined by an external auditor. The additional profit contribution II should then be shared equally by the two companies. While we have used this actual example for deriving inductively the mechanism for a fair allocation of profits, this result can also be derived deductively. The objective (14) may be reformulated as:

$$[\text{profit (Coop)} \cdot \text{share}_A - \text{profit}_A(\text{Non-coop})] \cdot [\text{profit(Coop)} \cdot (1\text{-share}_A) - \text{profit}_B(\text{Non-coop})] \Rightarrow \max \tag{21}$$

Taking the derivative of (21) with respect to the share of company A (share_A) and setting it to zero gives:

$$\text{Profit(Coop)}^2 - 2 \cdot \text{profit(Coop)}^2 \cdot \text{share}_A - \text{profit(Coop)} \cdot \text{profit}_B(\text{Non-coop}) + \text{profit(Coop)} \cdot \text{profit}_A(\text{Non-coop}) = 0 \tag{22}$$

Solving (22) with respect to the share results in:

$$\text{Share}_A = ½ + [\text{profit}_A(\text{Non-coop}) - \text{profit}_B(\text{Non-coop})]/[2 \cdot \text{profit(Coop)}]. \tag{23}$$

In this paper we have solved the various nonlinear optimization problems with a nonlinear optimization software that is implemented in the SOLVER which is provided as an add-in to the spreadsheet program EXCEL by Microsoft. If a company does not have access to such a program it can use a heuristic to determine a solution that comes close to the optimal Nash solution. As a first step, it has to predict the sales volume resulting from a joint salesforce as given by equation (24).

$$S_i^{joint} = S_i^{current} \cdot \left(\frac{m \cdot \sum_{j \in A,B} V_j}{V_i} \right)^{\beta_i} \qquad (i \in A, B) \tag{24}$$

Based on these values, the subsidiary operating the joint salesforce can determine the optimal joint salesforce budget by using equation (25).

$$\tilde{V} = \beta_A \cdot g_A \cdot S_A^{joint} + \beta_B \cdot g_B \cdot S_B^{joint} \tag{25}$$

Finally, each company can separately determine the optimal advertising budget according to its own interest from equation (26).

$$W_i = \gamma_i \cdot g_i \cdot S_i^{joint} \qquad (26)$$

This study shows that the application of game theory gives valuable insights into the structure of a rule for allocating the profits of a joint salesforce across participating companies. Based on the assumption that the companies have already optimized their budgets we can infer the salesforce as well as advertising elasticities from sales and cost data. Based on these values, we show how to determine the optimal budgets in a cooperation or how to heuristically come close to the optimal solution. This is an important device for companies that want to cooperate with other companies with respect to their salesforces. All steps are illustrated with data of an actual case study which shows that other solutions than the Nash solution produce rather high asymmetric distributions of additional profit that make a cooperation very unattractive. Company A found this results very instructive and is currently negotiating a cooperation contract with company B. Of course, the procedure and results can be generalized to any number of cooperating companies.

6 Limitations

The purpose of this investigation was to show that with very few data on current sales and budgets at the aggregate company level it is possible to infer elasticities of sales with respect to changes of the salesforce as well as advertising budget. This information can, then, be used to calibrate a commonly used aggregate sales response function that provides a company with predictions of sales depending on the additional calling-time resulting from a cooperation of salesforces. Of course, this very operational procedure comes at the cost of very strong assumptions on sales response. First, we assume that the companies operate with their budgets at the optimal level. Although the company in our case study felt this assumption to be true and the elasticity values are in the range of empirically observed values, this is not a proof of the correctness of the values. However, if there is only data on sales and budget levels available or companies want to exchange only this type of data, which is the usual case, then there is no other choice than to apply this method and optimality assumption. Second, our response function implies constant elasticities while the values actually decrease with increasing budget in practice. Given that the elasticity values are not precise anyway this overestimation may be tolerable. Third, we rely on an aggregate assumption concerning the additional calling-time from a cooperation as operationalized in response function (7). This does not reflect the heterogeneity that may be given when investigating the problem at the account level. Unfortunately, with data on just sales and budgets at the company level only, it is not possible to take into account such effects. Nevertheless, if we want to support companies in typical decision situations with very limited data we have to work with assumptions in the spirit of Lodish [10]: It is better to be vaguely right than to be precisely wrong (by not utilizing a model at all).

7 Conclusions

Cooperating salesforces are advantageous for companies if the presentation of the joint assortment and the exclusive assignment of accounts to the salesforces lead to additional calling-time and as a result to more profit from sales. In this situation, companies that have to decide on a cooperation need a method to predict the effects of a cooperation on profit. Since data on response functions is often not available and even more not from potentially cooperating partners, we suggest a method to infer the elasticities from sales and cost data based on the assumption that budgets are optimal. An actual case study shows that a simple cooperation of separate salesforces may lead to asymmetric additional profits which may be perceived as unfair. Therefore, it is necessary to negotiate an allocation of profits that is considered to be fair by all participating companies. This is not a trivial task because the usual solution of allocating the costs on the basis of commissions on achieved sales volumes may also lead to asymmetric additional profits and may therefore not be acceptable. In this situation, we have applied results from game theory on cooperative two-person-games. We propose to use the Nash solution which has proven to be the only one meeting criteria of fairness across cooperating players. According to this solution, it is optimal to allocate parts of the profit in advance, namely the level of profit contribution II of the current situation, and to then share the remaining profit equally between the participating companies. Overall, we have suggested a very operational method for predicting the effects of a cooperation of two salesforces. We have also derived a rule for the allocation of profits that can be considered fair for the cooperating players. These results are currently applied by a company in negotiations for cooperations.

References

1. Albers S (1996a) Optimization Models for Salesforce Compensation, Eur J Oper Res 89: 1-17.
2. Albers S (1996b) CAPPLAN: A decision support system for planning the pricing and sales effort policy of a salesforce, Eur J Mark 30,7: 68-82.
3. Albers S (1998) A framework for analysis of profit contribution variance between actual and plan, Intern J of Res Mark 15: 109-122.
4. Anderson E (1988) Strategic implications of Darwinian economics for selling efficiency and choice of integrated or independent sales forces, Man Sc 34: 599-618.
5. Assmus G, Farley JU, Lehmann DR (1984) How advertising affects sales: Meta analysis of economic results, J of Mark Res 21: 65-74.
6. Churchill GAjr, Ford NM, Walker OC (1997) Sales force management, 5th ed., Irwin: Chicago et al.
7. Dorfman R, Steiner PO (1954) Optimal advertising and optimal quality. Am Econ Rev 44: 826-836.
8. Hanssens DM, Parsons LJ, Schultz RL (1990) Market response models: Econometric and time series analysis, Kluwer: Boston/Dordrecht/London.
9. Horsky D, Nelson P (1996) Evaluation of salesforce size and productivity through efficient frontier benchmarking, Mark Sc 15: 301-320.
10. Lodish LM (1974) 'Vaguely right' approach to sales force allocations, Harv Bus Rev 52,1: 199-124.
11. Nash JF (1950) The bargaining problem. Econometrica 18: 155-162.
12. Nash JF (1953) Two-person cooperative games. Econometrica 21: 128-140.
13. Rangaswamy A, Sinha P, Zoltners A (1990) An integrated model-based approach for sales force structuring, Mark Sc 9: 279-298.
14. Rao RC, Turner RE (1984) Organization and effectiveness of the multiple-product salesforce, J of Pers Selling & Sales Man 4,1: 24-30.
15. Skiera B, Albers S (1998) COSTA: Contribution optimizing sales territory alignment, Mark Sc 17: 196-213.