Inverse Statistics in Tehran Price Index

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Abstract

We present an investigation on the time series of TEPIX and S&P500, former as an emerging market and later as an efficient one, using the inverse statistic approach [1,2]. Specifically by graphing the distribution of waiting time (also called investment horizon) of TEPIX and S&P500, we compare the behavior of these two kinds of stock market. Also by performing mentioned analysis on "White noise" as a process which contains no information, we report a similarity between the investment horizon PDF of S&P500 and White noise and a non-similarity between the investment horizon PDF of TEPIX and White noise.

Key words: Inverse statistics, Econophysics, Interdisciplinary physics

1 Introduction

Analyzing financial data using statistical mechanics tools and methods is a new and interesting trend in physics and especially in statistical mechanics and complexity. The every moment generating data let the scientist of statistical mechanics to make an empirical test on their theories and methods and use their statistical instruments to analysis these data. The statistical analysis of economic or financial time series used to be done by studying the distribution of returns. that is the distribution of log-difference of prices over a time interval Δt and includes the statistical features of price changes in an special period of time. But one important question is that in an initial investment made at time t, aiming a certain level of gain, what is the most likely waiting time? In order to answer this question, inverse statistics suggest to inverse the fixed (waiting time) and fluctuating variables (returns) so that study the distribution of waiting time needs to reach to a certain level of return instead of studying distribution of returns in an special waiting time. The maximum

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of this distribution signifies the optimal investment horizon for an investor aiming for a given return. In other word, letting S(t) as the asset price, the logarithmic return at time t, calculated over a time interval Δt, is:

$$r_{\Delta t}(t) = lnS(t+\Delta t) - lnS(t), \tag{1}$$

Then the investment horizon is defined as the time $\tau(t) = \Delta$t so that the inequality $r_{\Delta t}(t) \geq \rho$ when ,$\rho \geq 0$ or $r_{\Delta t}(t) \leq \rho$ when $\rho \leq 0$, is satisfied for the first time. The investment horizon distribution, $p(\tau_\rho)$, is then the distribution of investment horizons τ_ρ (see Fig. 2,3 and 4) averaged over the data.

A classic assumption made in theoretical finance is that the asset prices follow a geometrical Brownian motion, i.e. s(t) = ln S(t) is just a Brownian motion. For a Brownian motion, the investment horizon (first passage time) problem is known analytically [13–15]. It was shown that the investment horizon distribution is given by the Gamma-distribution:

$$p(t) = \frac{\nu}{\Gamma(\alpha/\nu)} \frac{|\beta|^{2\alpha}}{(t+t_0)^{\alpha+1}} exp\{-(\frac{\beta^2}{(t+t_0)})^{\nu}\} \tag{2}$$

Which have been found in [16] that, for large τ_ρ, $p(\tau_\rho)$ scales as:

$$p(t) \simeq \tau_\rho^{-\alpha} \tag{3}$$

With $\alpha = 2 - H$ and the hurst exponent is found 0.74 ± 0.02 and 0.46 ± 0.02 respectively for TEPIX and S&P500 [7]. Furthermore, the maximum of this distribution, i.e. the optimal investment horizon, is located at:

$$\tau_\rho^* = \beta^2(\nu/(\alpha+1))^{1/\nu} - t_0 \tag{4}$$

For a given level of return ρ. If the underlying asset price process is geometric Brownian, then one would have $\tau^* \sim \rho^2$ for all values of ρ which have been seen in [3] that is far from what is observed empirically. It is well-known that many historic financial time series posses an (often close to exponential) positive drift over long time scales. If such a drift is present in the analyzed time series, one can obviously not compare directly the histograms for positive and negative levels of return. In this paper we will be interested in making such a comparison, one has to be able to reduce the effect of the drift significantly. One possibility for detrending the data is to use deflated asset prices.

In this paper we follow up on these studies and investigate the corresponding statistical distributions for the Tehran price index and S&P500. In particular, this work focuses on comparing the results of the above analysis for the two kinds of stock markets. In order to get more details we perform the same analysis for White noise and study the behavior of investment horizon PDF of these three sets of data.

2 Analysis of the TEPIX and S&P500 stock markets

As our scenario is based on the comparison between the two stock markets, we perform the above analysis on the daily TEPIX and S&P500 data for a period of 4 years and compare the result with which is known as a short scale of time. We begin by graphing the values of the fluctuations as a function of time. Figs 1. shows the time series of daily returns for the TEPIX and S&P500. The difference between the daily trading volume of the two kinds of stock market is obvious from this figure.

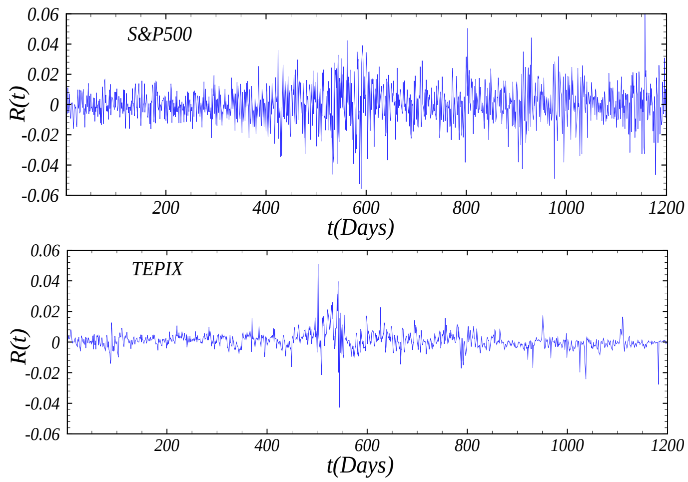

Fig. 1. Time Series of return for S&P500 and TEPIX.

In Figs 2. waiting time (*investment horizon*) distribution τ for the TEPIX and S&P500 is shown for 1200 daily data which is smoothed by FFT filter and adjacent averaging. The return level used to obtain these results is $\rho = \pm\sigma$, where σ is the standard deviation of the one-day returns of each of these two stock markets. From this figure we can see that there are a notable difference in the position of positive and negative diagrams for these two kinds of stock market. The distribution for the positive return is obviously upper than such distribution for the same but negative level of return. This indicates that in TEPIX, in a same period of time, the probability of reaching to a positive level of return (gain) is more than the respective probability for the same but negative level of return (loos), It's completely inverse in S&P500 which is known as an efficient market. This Observation leads to the fact that hopes for gaining in TEPIX is more than loosing which is vice Versa for S&P500. To confirm our findings, the same work have been done for $\rho = \pm 3\sigma$ in Figs 3.

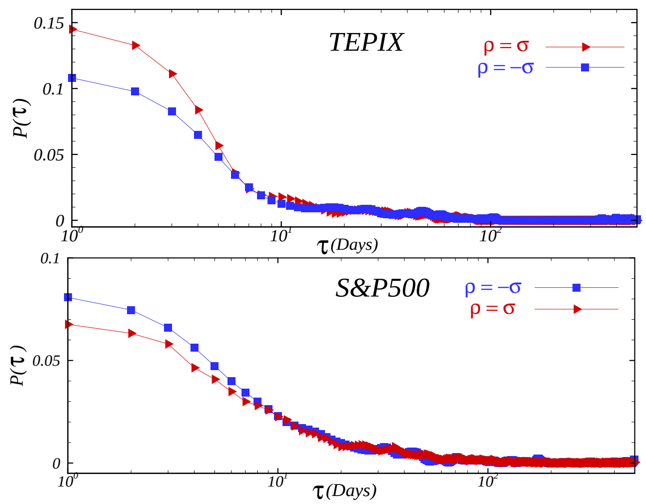

Fig. 2. The probability distribution of *normalized* waiting time τ needed to reach return levels $\rho = -\sigma$ and $\rho = \sigma$ for the TEPIX and S&P500.

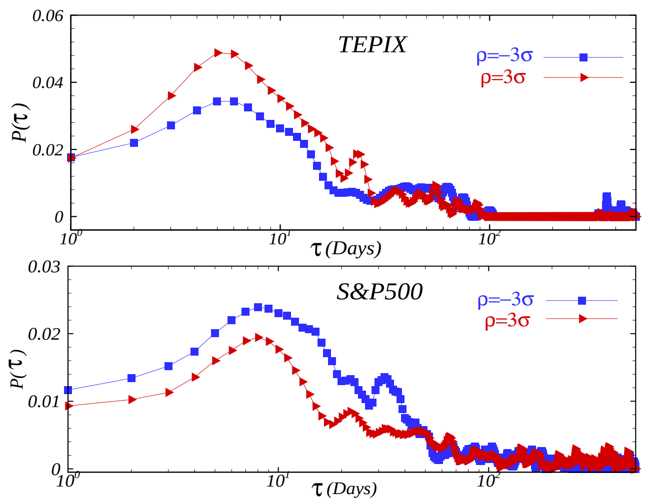

Fig. 3. The probability distribution of *normalized* waiting time τ needed to reach return levels $\rho = -3\sigma$ and $\rho = +3\sigma$ for the TEPIX and S&P500.

Specifically to compare the investment horizon of the mentioned stock markets with White noise for a same level of return, we extended the analysis for level of return $\rho = 0.04$ in fig 4. This figure shows that (i) there is a remarkable similarity between the diagrams of S&P500 and White Noise. This interestingly indicates that there might be no information among this analysis on the financial markets and this market data behaves just as a random data. (ii) for short waiting times, TEPIX and S&P500 follow same behavior. (iii) the TEPIX distribution of waiting time goes upper than for the S&P500 for larger time intervals. In the view of this, we gets that during a certain period of time, especial change in price for TEPIX is more likely than for S&P500. And (iv) the maximum of S&P500 waiting time distribution occurs sooner than of the TEPIX. This indicates that S&P500 implies a bull market while TEPIX implies bear market, which means that the most probable span of time needs to reach to an aiming gain for S&P500 is shorter than what is for TEPIX.

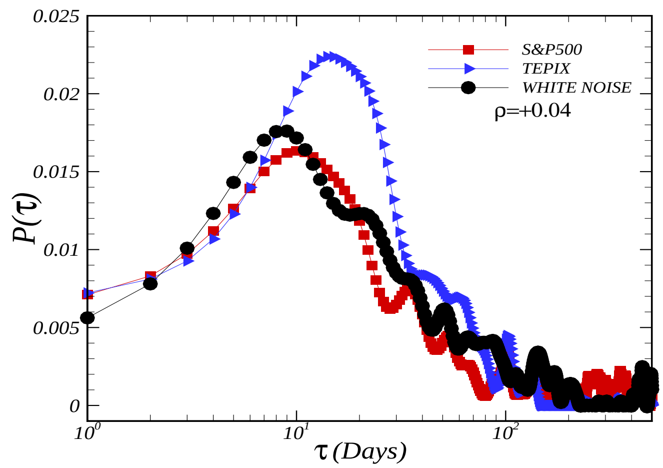

Fig. 4. The probability distribution of *normalized* waiting time τ needed to reach return level $\rho = 0.04$ for the TEPIX, S&P500 and WHITE NOISE.

In addition, we have however also shown for comparison the optimal investment horizon of these two stock markets and White noise. Fig 5. illustrates the mentioned similarity between S&P500 and White noise and also shows that the two stock markets have similar behavior for short time scales which are in agreement with the results of our observations from the previous figures.

3 Conclusions

In conclusion, we have employed the inverse statistics method to investigate the data for the TEPIX and S&P500. One of our interesting observations was the inversion of gain and loos distributions of TEPIX and S&P500. The larger level of the curve of gain distribution in TEPIX corresponds to a feature of an emerge market and lower level of it corresponds to efficiency of S&P500. The other important results of our comparative discussion belongs to the remarkable similarity in the behavior of investment horizon distribution of S&P500 and White noise. As it was mentioned before, white noise is known as a random data and is an unpredictable process. Therefor just if the behavior a data set deviates from White noise, studying it's fluctuations will uncover

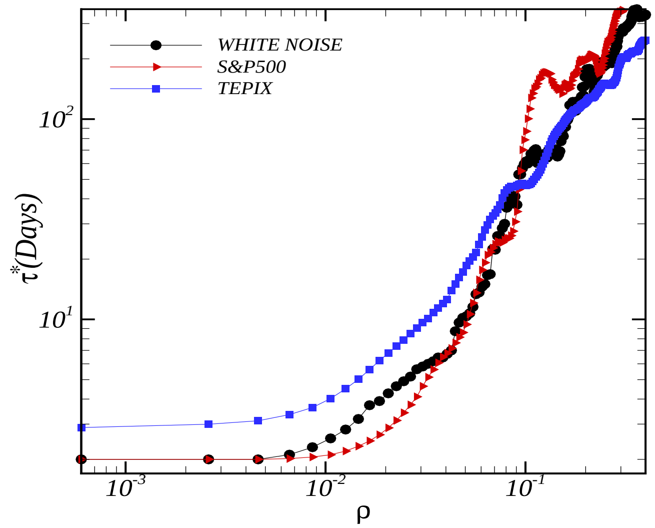

Fig. 5. The optimal investment horizon τ^* of TEPIX (Square), S&P500 (Triangle) and WHITE NOISE (Circle) for levels of return ρ.

facts about it. According to this, looking at Fig 4. and Fig 2. may leads us to the conclusion that by using inverse statistics approach for S&P500 as an efficient market, eventually we can get no information. On the other hand, the deviation of TEPIX distribution of investment horizon from White noise indicates that this analysis concludes information for non-efficient markets.

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