Analysis of Interest Rate Curves Clustering Using Self-Organising Maps

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Abstract. The paper presents the analysis of temporal evolution of interest rate curves (IRC). IRC are considered as objects embedded in a high-dimensional space composed of 13 different maturities. The objective of the analysis was to apply a nonlinear nonparametric tool (Self-Organising Maps) to study the clustering of IRC in three different representations: the original curves, the increments and 3-parametric Nelson-Siegel model. An important finding of this study reveals the temporal clustering of IRC behaviour which is related to the market evolution. Other results include the relative analysis of CHF-EUR evolution and the clustering found in the evolution of factors used by Nelson-Siegel model. The analysis of the consistency of these factors to represent the typical IRC behaviour requires further work. Current results are useful for the development of interest rates forecasting models and financial risk management.

Keywords: interest rate curves, Self-Organising Maps, clustering, financial predictions

Introduction

Interest rate curve (IRC) is a fundamental object for economics and finance. By definition, the IRC is the relation between the interest rate (or cost of borrowing) and the time to maturity of the debt for a given borrower in a given currency. Figure 2 below presents some typical IRCs.

Interest rates depend on time and on maturity which defines the term structure of the interest rate curves. IRCs are composed of interest rates at different maturities which move coherently in time: evolutions of different maturities in time can not be considered independently – they follow some well known stylised facts (see, for example, [1] and [3]). In general, they have to be considered as functional data.

The present study deals with the analysis of interest rate curves temporal clustering. The main idea is to consider IRCs as integral objects evolving in time and to study the similarity and dissimilarity between them. Detection of finite number of clusters can help to reveal typical patterns and their relationships with market conditions.

In the present study Self-Organising (or Kohonen) Maps - SOM, are used in order to analyse and to model clustering structure of IR curves. The paper extends the research first time presented in [1] in the following directions: detailed analysis of IRC clustering and their daily increments, comparison of Swiss franc and Euro IRC, analysis of IRC clustering in a Nelson-Siegel parametric feature space.

Real data analysis is devoted to the exploration of Swiss franc (CHF) and Euro (EUR) interest rates. Daily data during several consecutive years are studied. The IRCs are composed of LIBOR interest rates (maturities up to 1 year) and of SWAP interest rates (maturities from 1 year to 10 years).

In the present study curves are considered as objects embedded into 13-dimensional space induced by the 13 interest rate levels of different maturities. IRC data are available on specialised terminals like Reuters and Bloomberg, and are provided for fixed time intervals (daily, weekly, monthly) and for some definite maturities (in this research we use daily data and maturities of 1 week, 1, 2, 3, 6 and 9 months; 1, 2, 3, 4, 5, 7 and 10 years). Evolution of CHF and EUR interest rates for different maturities are presented in Figures 1a and 1b.

a) b)

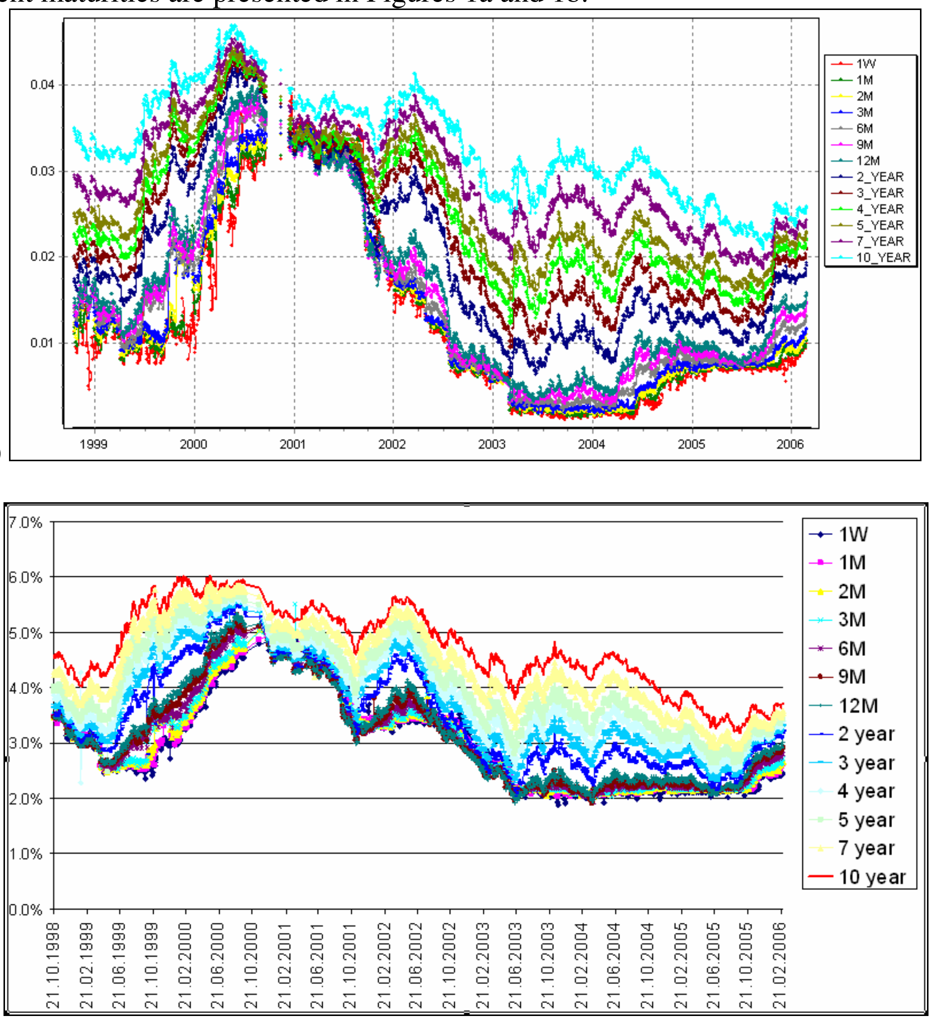
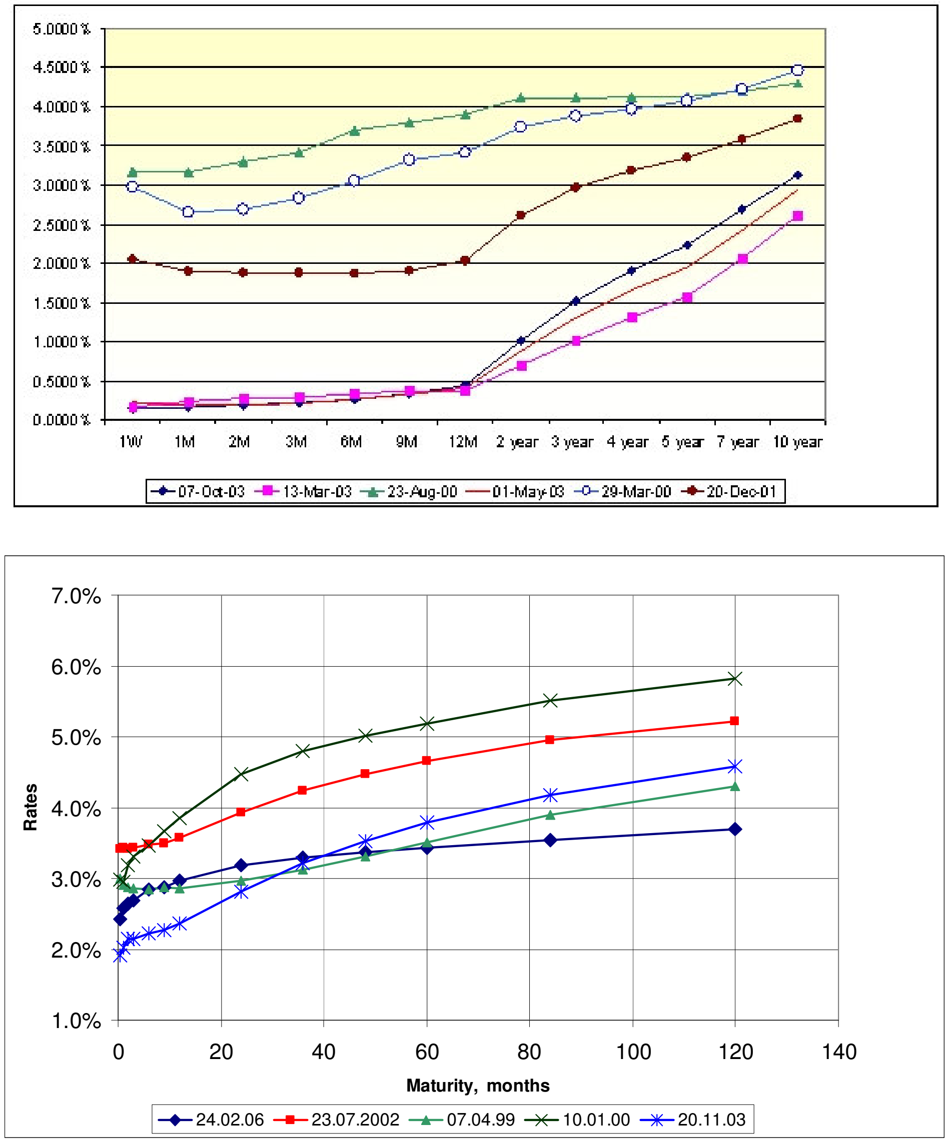


Figure 1. Time series of interest rates for different maturities: CHF (a) and EUR (b).

Typical curves for CHF and EUR are given in Figures 2a and 2b.



a) b)

Figure 2. Typical examples of IRCs for several days: CHF (a) and EUR (b).

The relationships between different maturities can be studied by using correlation matrix (see Figure 3, where CHF rates were used). Different situations can be observed: high linear correlations, nonlinear correlations, multi-valued relationships. Therefore it seems quite reasonable to apply nonlinear adaptive tools as Self-Organising Maps to study the corresponding patterns.

The results of linear analysis carried out using PCA are presented in Figure 4. First five components explain more than 90% of the variance.

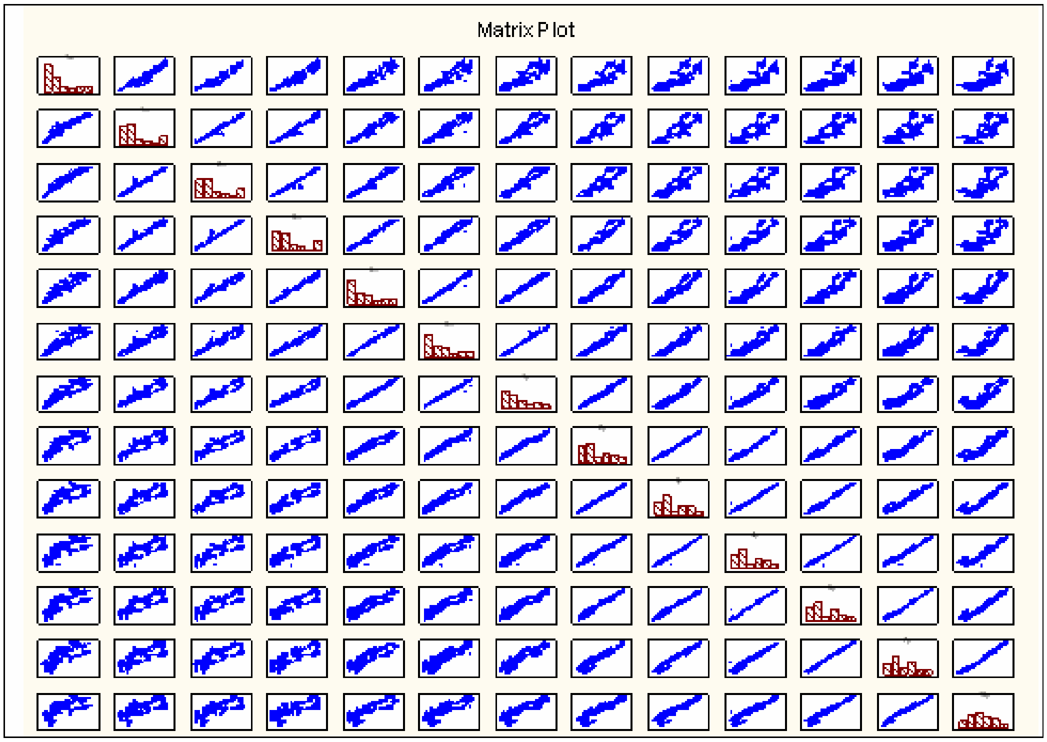


Figure 3. CHF interest rates correlation matrix.

An important and interesting approach complementary to classical empirical analysis of interest rates time series was developed in [4,5,6] where both traditional econophysics studies (power law distributions, long range correlations, etc.) and a coherent hierarchical structure of interest rates were considered in detail.

An empirical quantitative analysis of multivariate interest rates time series and their increments (carried out while not presented in this paper) includes the study of autocorrelations, cross-correlations between different maturities, detrended fluctuation analysis, embedding, analysis of distributions and tails.

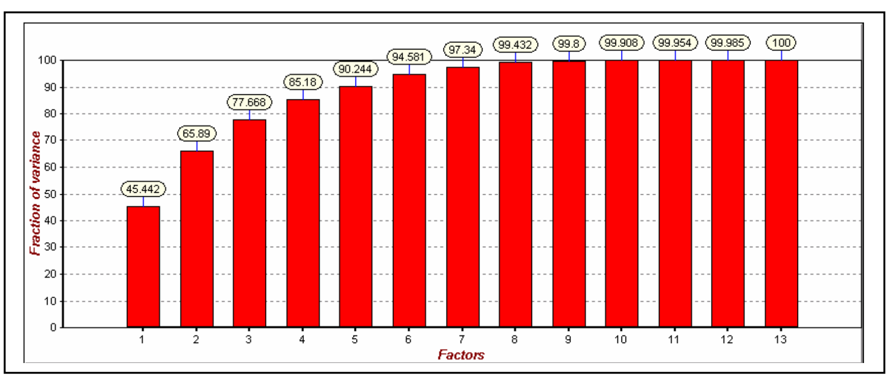


Figure 4. CHF interest rates. PCA analysis of IRC. The explained variance by the number of PCA components.

In the following section the theory of SOM and its application to the current study is briefly explained following [1, 7, 8].

Self-Organising Maps

Unsupervised learning

Self-Organising Maps belong to the machine learning algorithms dealing with unsupervised learning trying to solve clustering, classification or density modelling problems using unlabeled data. SOM are widely used for the dimensionality reduction and visualisation of high-dimensional data (projection into two-dimensional space). Unlabeled data are vectors in a high-dimensional feature space that have some attributes (or coordinates) but have no target values, neither continuous (regression) nor discrete labels (classification). The main task of SOM is to "group" or to "range" in some manner these input vectors and to try to catch regularities (or patterns) in the data preserving topological structure by using defined similarity measures. Detailed presentation of SOM or Kohonen maps along with a comprehensive review of their application, including socio-economic and financial data are given in [7, 8].

SOM network structure

Self-organising map is a single layer feedforward network where the output neurons are arranged in a two-dimensional topological grid. Type of the grid may be rectangular or hexagonal. In the first case each neuron (except borders and corners) has four nearest neighbours, in the second – six ones. So hexagonal map presents smoother result but requires a bit more calculations. Attached to every neuron there is a weight vector with the same dimensionality as the input space. Each unit $\boldsymbol{i}$ has a corresponding weight vector $\boldsymbol{w}_i = \{ w_{i1}, w_{i2}, \dots , w_{id} \}$ where $\boldsymbol{d}$ – is a dimension of the input feature space. In general, SOM is a projection of high-dimensional data into a low-dimensional (usually two) space using some similarity measures.

Learning algorithm

In general, learning is a procedure used to tune optimal parameters of the network. In case of SOM parameters are weights of the neurons. As it was mentioned above, these weights are their coordinates in the input feature space.

SOM is based on a *competitive* learning. It means that output neurons compete among themselves to be activated or fired. As a result only one output neuron wins. It is called *winner-takes-all* neuron or just *winning* neuron. Hence, the winning neuron $\boldsymbol{w_w}$ is a neuron which is the "closest" (in some metric) neuron to the input example $\boldsymbol{x}$ among all $\boldsymbol{m}$ others:

$$d(x, w_w) = \min_{1 \le j \le m} d(x, w_j) \quad (1)$$

SOM initialisation

First step of the SOM learning deals with choosing the initial values of the neurons' weights. There are two methods widely used for the initialisation of SOM [7].

1. Randomly selected $\boldsymbol{m}$ points from the data set equal to the number of neurons. Their coordinates are assigned as neurons' weights.
2. Small random values are sampled evenly from the input data subspace spanned by the two largest principal component eigenvectors. This method can increase the speed of the learning significantly because the initial weights already give good approximation of SOM weights.

Weights updating

The iteration training process for each ***i*** from ***m*** neurons is

$$w_i(t+1) = w_i(t) + h_i(t)[x(t) - w_i(t)] \qquad (2)$$

$h_i(t)$ **–** is a *neighbourhood function* (definition see below).

Neighbourhood function

We should define a so-called *neighbourhood* function $h_i(t)$. It is a function of time ***t*** (training iteration). It defines the neighbourhood area of the neuron ***i***. The simplest of them refers to a neighbourhood set of array points around the node ***i***. Let this index set be denoted ***R***, whereby

$$\begin{cases} h_i(t) = \alpha(t) & if \quad i \in R \\ h_i(t) = 0 & if \quad i \notin R \end{cases} \qquad (3)$$

where $\alpha(t)$ is a *learning rate* defined by some monotonically decreasing function of time and $0 < \alpha(t) < 1$.

Another widely used neighbourhood function can be defined as a Gaussian

$$h_i(t) = \alpha(t)\exp\left(\frac{d(i,w)}{2\sigma^2(t)}\right) \qquad (4)$$

where $\sigma(t)$ is a width of the kernel (corresponding to ***R***) is a monotonically decreasing function of time as well.

Exact forms of $\alpha(t)$ and $\sigma(t)$ are not critical. They should monotonically decrease in time and can be even linear. They decrease to zero value for $t \to T$, where ***T*** – total number of iterations. For example, $\alpha(t) = c\,(1-t/T)$, where ***c*** is a predefined constant (*T/100*, for example).

As a result, at the beginning when the neighbourhood is broad (cover all neurons), the self-organising takes place on the global scale. At the end of training, the neighbourhood has shrunk to zero and only one neuron updates its weights.

SOM visualisation tools

Several visualisation tools ("maps") are used to present trained SOM and to apply it for data analysis:

- ***Hits*** map – how many times (number of hits) each neuron wins (depends on presenting data set);
- ***U-matrix*** (***u***nified distance ***matrix***) – map of distances between each of neurons and all his neighbours. It is particularly useful for detailed analysis.
- ***Slices*** – 2D slices of the SOM weights (total number of maps is equal to the dimension of the data);
- ***Clusters*** – map of recognised clusters in the SOM.

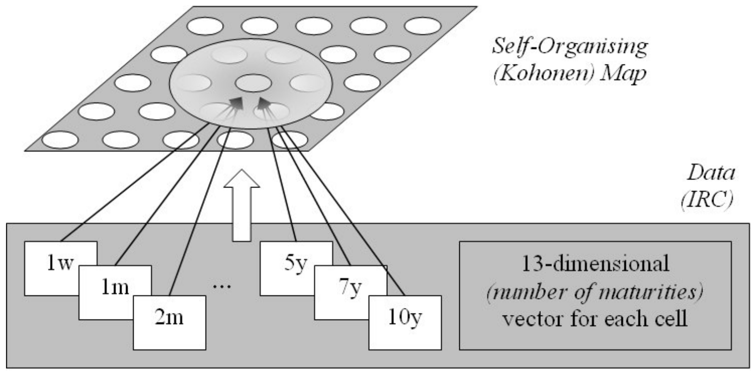


Figure 5. SOM structure for IRC analysis using SOM.

Temporal dimension (date) of IR data was not directly taken into account. Moreover, a priori it is not known how many clusters can be detected. Comprehensive analysis was carried out considering different number of potential clusters. Structures of clusters, their properties and relationships between them are examined in details.

For the completeness of the research the SOM analysis was performed also on: 1) temporal increments of interest rates and 2) in a feature space characterized by a well known 3 factors (level, slope, and curvature) Nelson-Siegel model [2].

Interesting findings deal with the observation of several typical behaviours (clusters) of IR curves and their clustering in time according to different market conditions: low rates, high rates, and periods of transition between the two. Such analysis is an important nonlinear exploratory tool and can help in prediction of interest rates curves.

Results and discussions

The SOM structure used in this study is illustrated in Figure 5. The detailed results on the clustering of CHF interest rate curves were presented in [1]. The results below present the new findings following the mentioned study.

To reveal the relations in the evolution of Euro and CHF IRC, the analysis scheme of [1] was applied to IRC of both currencies for the same maturities and time periods using the same structure of SOM. The U-matrix and the clusters obtained with k-means applied for the latter are presented in Figure 6.

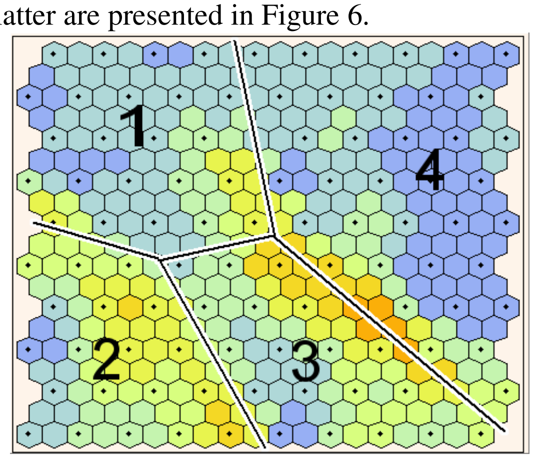


Figure 6. U-matrix of the Self-Organizing Map used for EUR IRC clustering. The boundaries of four clusters found by k-means are visualized.

The visualization of these clusters in the temporal domain reveals the well-defined temporal clustering detected in the evolution of EUR IRC. These results are presented in Figure 7, where the clusters are visualized with bold grey dots over the curves. For a direct comparison of CHF and EUR clusters, CHF clusters found in [1] are presented with black dots, reflecting the similar behaviour of the currencies. At the same time, the transitional periods (changes from cluster to cluster) are different, providing the temporal delay in switching the cluster (a typical style of behaviour) by IRC in different currencies.

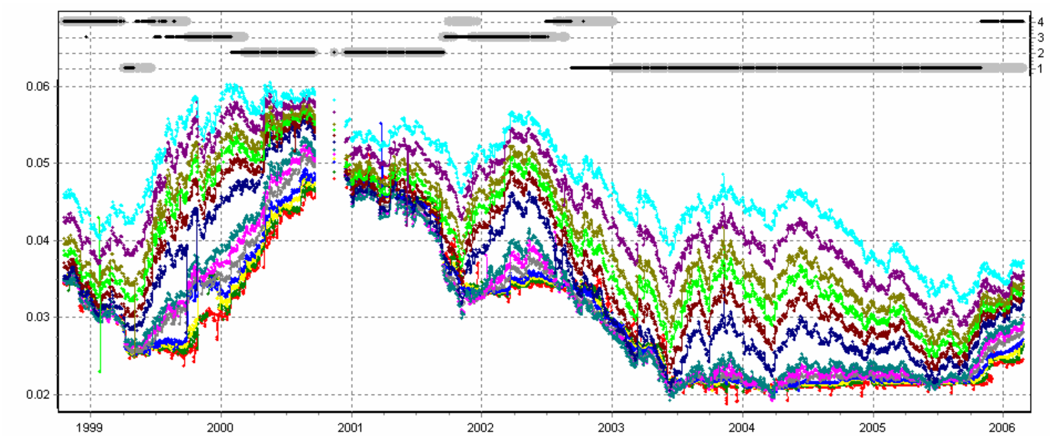


Figure 7. EUR interest rates. Black dots – clusters found in CHF, bold gray dots are clusters in EUR. Except the periods of the end of 2002 and from end 2005 to beginning 2006, the clustering structure is very similar.

The analysis of the daily *increments* of CHF interest rates was carried out for the period of 2001-2006. The correlations between the interest rates of different maturities (Figure 3) and their increments (Figure 8) appeared to be of considerably different structure. No significant correlation between the increments for short-term maturities was observed. Next, the curves composed of interest rates increments were analysed using the Self-Organizing Maps following the scheme presented in [1]. No temporal clustering of these curves was observed (Figure 9). Black dots above time series of the increments correspond to the found clusters.

	R1w	R1m	R2m	R3m	R6m	R9m	R1y	R2y	R3y	R4y	R5y	R7y	R10y
R1w	1.00	0.39	0.32	0.31	0.23	0.16	0.18	0.11	0.11	0.09	0.07	0.04	0.01
R1m	0.39	1.00	0.44	0.38	0.26	0.23	0.20	0.19	0.17	0.15	0.14	0.12	0.07
R2m	0.32	0.44	1.00	0.49	0.35	0.26	0.24	0.20	0.18	0.16	0.15	0.12	0.10
R3m	0.31	0.38	0.49	1.00	0.44	0.34	0.31	0.27	0.24	0.22	0.20	0.18	0.12
R6m	0.23	0.26	0.35	0.44	1.00	0.65	0.56	0.40	0.39	0.36	0.34	0.30	0.25
R9m	0.16	0.23	0.26	0.34	0.65	1.00	0.69	0.53	0.50	0.48	0.45	0.41	0.32
R1y	0.18	0.20	0.24	0.31	0.56	0.69	1.00	0.61	0.60	0.57	0.55	0.49	0.41
R2y	0.11	0.19	0.20	0.27	0.40	0.53	0.61	1.00	0.95	0.92	0.88	0.80	0.68
R3y	0.11	0.17	0.18	0.24	0.39	0.50	0.60	0.95	1.00	0.97	0.95	0.89	0.77
R4y	0.09	0.15	0.16	0.22	0.36	0.48	0.57	0.92	0.97	1.00	0.98	0.92	0.81
R5y	0.07	0.14	0.15	0.20	0.34	0.45	0.55	0.88	0.95	0.98	1.00	0.95	0.84
R7y	0.04	0.12	0.12	0.18	0.30	0.41	0.49	0.80	0.89	0.92	0.95	1.00	0.88
R10y	0.01	0.07	0.10	0.12	0.25	0.32	0.41	0.68	0.77	0.81	0.84	0.88	1.00

	R1w	R1m	R2m	R3m	R6m	R9m	R1y	R2y	R3y	R4y	R5y	R7y	R10y
R1w	1.00	0.39	0.32	0.31	0.23	0.16	0.18	0.11	0.11	0.09	0.07	0.04	0.01
R1m	0.39	1.00	0.44	0.38	0.26	0.23	0.20	0.19	0.17	0.15	0.14	0.12	0.07
R2m	0.32	0.44	1.00	0.49	0.35	0.26	0.24	0.20	0.18	0.16	0.15	0.12	0.10
R3m	0.31	0.38	0.49	1.00	0.44	0.34	0.31	0.27	0.24	0.22	0.20	0.18	0.12
R6m	0.23	0.26	0.35	0.44	1.00	0.65	0.56	0.40	0.39	0.36	0.34	0.30	0.25
R9m	0.16	0.23	0.26	0.34	0.65	1.00	0.69	0.53	0.50	0.48	0.45	0.41	0.32
R1y	0.18	0.20	0.24	0.31	0.56	0.69	1.00	0.61	0.60	0.57	0.55	0.49	0.41
R2y	0.11	0.19	0.20	0.27	0.40	0.53	0.61	1.00	0.95	0.92	0.88	0.80	0.68
R3y	0.11	0.17	0.18	0.24	0.39	0.50	0.60	0.95	1.00	0.97	0.95	0.89	0.77
R4y	0.09	0.15	0.16	0.22	0.36	0.48	0.57	0.92	0.97	1.00	0.98	0.92	0.81
R5y	0.07	0.14	0.15	0.20	0.34	0.45	0.55	0.88	0.95	0.98	1.00	0.95	0.84
R7y	0.04	0.12	0.12	0.18	0.30	0.41	0.49	0.80	0.89	0.92	0.95	1.00	0.88
R10y	0.01	0.07	0.10	0.12	0.25	0.32	0.41	0.68	0.77	0.81	0.84	0.88	1.00

Figure 8. Correlation matrix of increments (%) for all maturities. Highlighted cells: correlation >0.5 (left); and >0.9 (right).

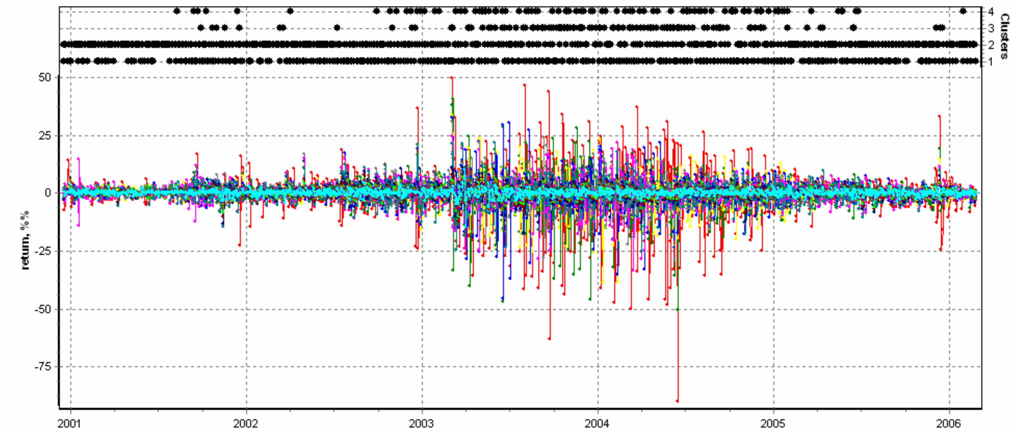


Figure 9. Increments (%) for all maturities, from 2001 to 2006. 4 clusters, no (visible) structures in time.

The same analysis was carried out using the squared increments (Figure 10). Though no clusters clearly related to the temporal evolution were found, the noticeably different behaviour during the period of high volatility can be recognised.

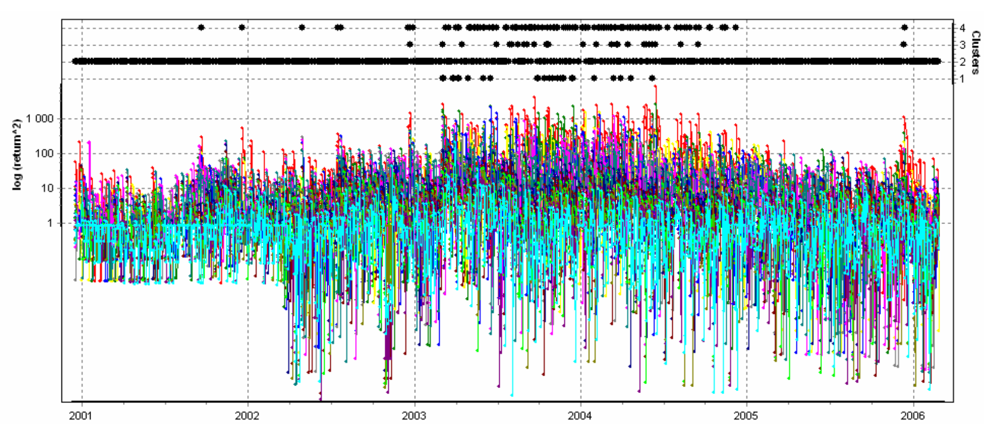


Figure 10. Squared Returns in %%, visualisation in log scale (SOM was trained with linear scale) from 2001 to 2006. Base cluster (stable behaviour) is number 2, but from 2003 to end of 2004 some unstable behaviour (mixture of clusters).

Finally, the feature space composed of Nelson-Siegel model factors [2, 3] was investigated with the proposed methodology. The Nelson-Siegel model is based on 3 factors corresponding to the long-term, short-term and medium term IR behaviour. These parameters can be interpreted in terms of level [(maturity 10 years)], slope [(maturity 10 years) – (maturity 3 months)] and curvature [2*(maturity 2 years) – (maturity3 moths) – (maturity 10years)]. These factors are often used to characterise IRC and for forecasting [3].

The application of SOM resulted in the following clustering model, presented as a SOM U-matrix in Figure 11.

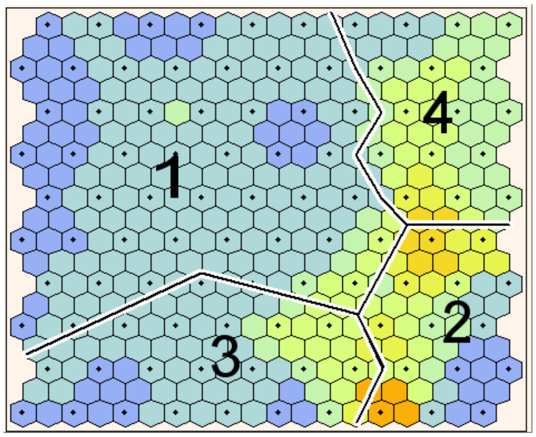


Figure 11. U-matrix of the Self-Organizing Map used for clustering of Nelson-Siegel model factors. The boundaries of four clusters found by k-means are visualized.

Temporal evolution of three parameters along with four clusters evolution (black dots) are presented in Figure 12. Clusters are well separated in time. The detailed analysis using SOM with different number of clusters confirmed that the selected number of clusters (4 clusters) qualitatively well explains most of the similarities and dissimilarities of the curves.

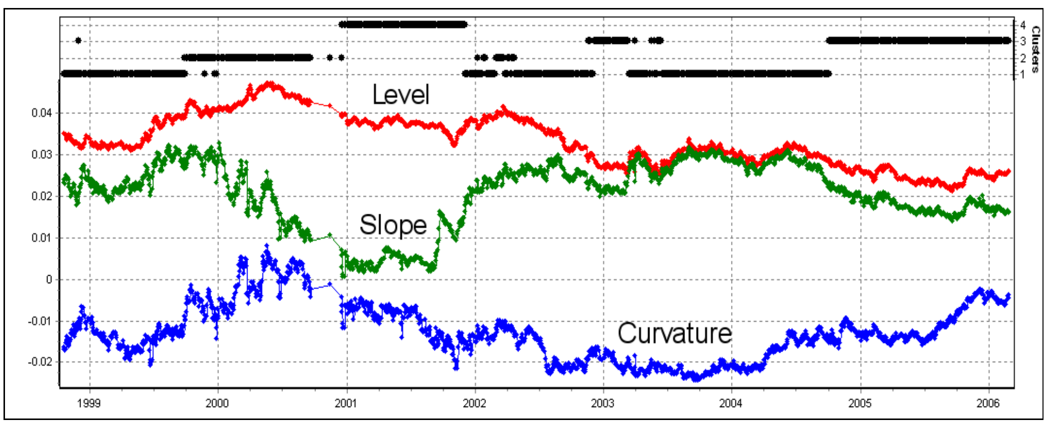


Figure 12. Time series and 4 clusters evolution of Nelson-Siegel factors.

Table 1. Qualitative description of the detected Nelson-Siegel clusters in respect to the range of model parameter values.

Variable / Cluster	1	2	3	4
Level	-	-	Low	High
Slope	High	Low	-	-
Curvature	Low	-	-	High

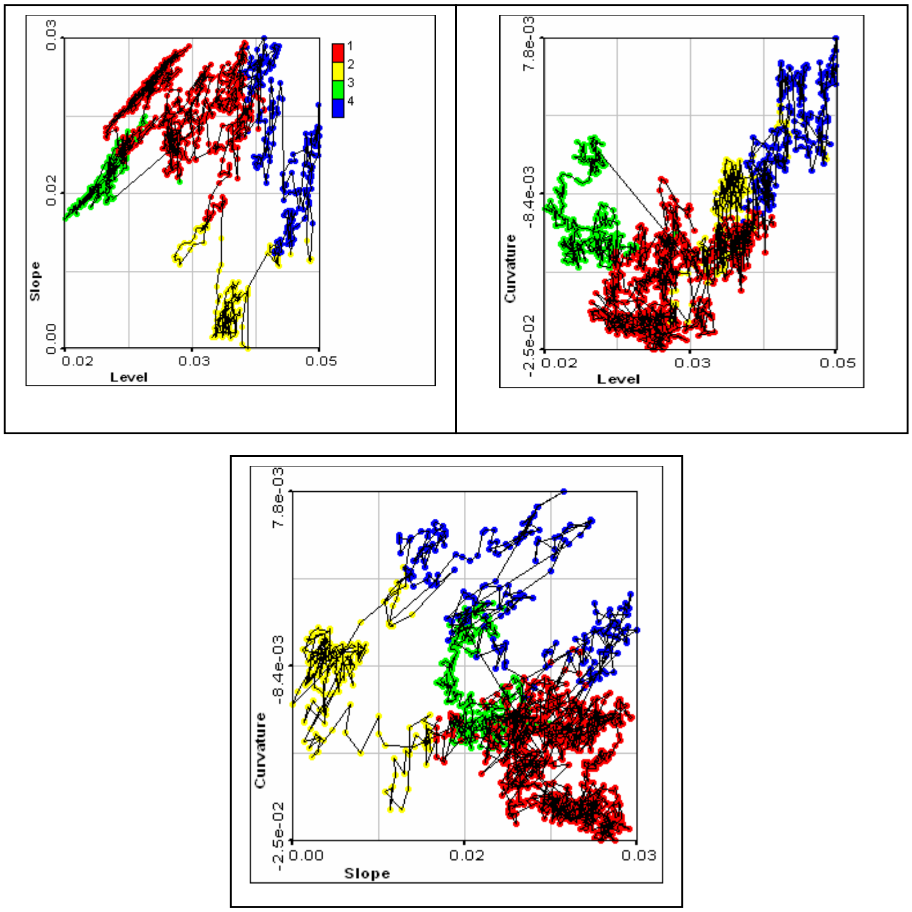


Figure 13. CHF IRC clusters trajectories presented on Nelson-Siegel factors.

From Figures 12 and 13 it follows that three factor space of Nelson-Siegel model provide a description of IRC evolution which possesses some temporal clustering, or period of typical behaviour. The qualitative description of the clusters is presented in Table 1, where "high" means that the corresponding factors (either level, slope or curvature) are in the area of high values, and "low" if in the low ones. However, the relation of the temporal clusters obtained for raw IRC curves (full 13-dimensional representation) to the ones obtained in the space of Nelson-Siegel parameters is not evident and requires further work.

Conclusions

Self-Organising (Kohonen) Maps were applied to study CHF and EUR interest rate curves clustering in time. The same analysis was carried out using daily increments and three factor Nelson-Siegel model. Interesting finding deals with the observation of several typical behaviours of curves and their clustering in time around low level rates, high level rates, and periods of transition between the two. Such analysis can help in the prediction of interest rate curves, evaluation of financial products and in financial risk management. Analysis carried out using three factor feature space confirms clustering structure and its temporal evolution, and its relation to the evolution of IRC provides interesting directions for further work.

Acknowledgements

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