

Economic cycles and their synchronization: spectral analysis of GDP time series from Italy, the Netherlands, and the UK

Lisa Sella^a, Gianna Vivaldo^b, Michael Ghil^{c,d}, Stéphane Hallegatte^{e,f}

Abstract: The present work applies several advanced spectral methods to the analysis of GDP fluctuations in Italy, the Netherlands, and the United Kingdom. These frequency-domain tools allow one to spectrally decompose, as well as reconstruct GDP time series from the data; they provide therewith a precise quantitative description of the series' main oscillatory components. These methods are well adapted to the analysis of short and noisy data, like the GDP time series analyzed herein; while fairly well known by now in the geosciences and life sciences, they are still not widespread in quantitative economics. We find very similar medium-size, short-size and near-annual cycles in all three GDP time series. These results agree fairly well with solutions of a non-equilibrium dynamic model (NEDyM) that introduces investment dynamics with adjustment delays and non-equilibrium effects into a neoclassical Solow growth model (Hallegatte, Ghil, Dumas, & Hourcade, 2007). The paper concludes with some remarks about the synchronization of economic fluctuations within Euro-area and outside it.

Keywords: Economic Cycles; Synchronization; Spectral Analysis; Non-Equilibrium Dynamical Models; Advanced Time Series Analysis.

Acknowledgments: We would like to thank Profs. Pietro Terna and Vittorio Valli for their valuable guidance of the lead author's doctoral work, and the latter also for many valuable suggestions about the economic interpretation of this work's results.

^a Dept. of Economics "S. Cagnetti de Martiis", University of Turin, Italy.

^b Dept. of Physics "A. Avogadro", University of Turin, Italy.

^c Geosciences Department and Environmental Research and Teaching Institute, Ecole Normale Supérieure, Paris, France.

^d Department of Atmospheric and Oceanic Sciences and Institute of Geophysics and Planetary Physics, University of California, Los Angeles, CA 90095-1565, USA.

^e Centre International de Recherche sur l'Environnement et le Développement, Nogent-sur-Marne, France.

^f Ecole Nationale de la Météorologie, Météo-France, Toulouse, France.

1. Introduction

Aggregate fluctuations are one of the most controversial topics in Macroeconomics: since the first systematic time series analysis on France, England, and the US datasets (Juglar, 1862), a recurrent behavior of economic crises has been hypothesized, with strong interdependence between boom and recession phases. From then on, different schools of economic thought debated on the nature and causes of economic fluctuations, each one putting forward its own interpretation and models about the fundamental characteristics and mechanisms of the cycles (Stanca, 2001; Arnold, 2002). However, the issue has not been finally resolved yet, in particular concerning the exogenous or endogenous nature of economic fluctuations (Slutsky, 1927; Frisch, 1933; Kaldor, 1940; Hicks, 1950; Goodwin, 1967; Kydland & Prescott, 1982; Long & Plosser, 1983; King & Rebelo, 2000; Chiarella, Flaschel, & Franke, 2005).

During the last twenty years the debate has been enriched by the diffusion of complexity studies in Economics: economic systems are increasingly thought of as complex adaptive dynamic systems (Arthur, Durlauf, & Lane, 1997), with the consequent spreading of new techniques and modeling tools. In this framework, Hallegatte, Ghil, Dumas, & Hourcade (2007) propose a non-equilibrium dynamic model (NEDyM), which introduces investment dynamics with adjustment delays and non-equilibrium effects into a neoclassical Solow growth model. Simulations on NEDyM show model dynamics with Keynesian features in response to exogenous shocks, thus producing endogenous business cycles with some recurrent periodicities.

Our work is an empirical inquiry of economic cycles by spectral analysis of GDP for three different European countries: Italy, the UK, and the Netherlands. This choice has been motivated by the characteristics of these time profiles: the UK series covers 53 years (1955:01-2008:01) of a quite large European economy outside the Eurozone, while the much shorter series for Italy (26 y, 1981:2-2007:1) and the Netherlands (31 y, 1977:01-2007:04) respectively represent a medium- and a small-size economy in Euroland. Thus, we compare countries of different economic magnitude both inside and outside the Euro-area in order to inspect their common cyclical movements. This allows us to reflect upon business cycles co-movements and synchronization, focusing on their possible causes and particularly on the effects of economic and monetary integration within EMU (cf. Bergman, 2007).

About this point, there is still no accord among scholars, neither from a theoretical nor from an empirical point of view. As an example, Krugman (1993) and Kalemli-Ozcan, Sørensen, & Yosha (2001) suggest that economic and capital integration leads to more specialized production structures and expands trade, thus reducing the synchronization of business fluctuations. On the contrary, Coe & Helpman (1995) and Frankel & Rose (1998) argue that the removal of trade barriers eases the transmission of demand shocks across countries, thus causing more symmetric structural shocks together with knowledge and technological spillovers, which in turn lead to more synchronized national business cycles. This evident ambiguity has not been resolved by empirical inquiries: many authors focalize on the role of the exchange rate variability, some suggesting an opposite relation between it and synchronization (Fatás, 1997; Dickerson, Gibson, & Tsakalotos, 1998; Rose & Engel, 2002), others a positive one (Inklaar & De Haan, 2001; De Haan, Inklaar, & Sleijpen, 2002), and a few no relation at all (Baxter & Stockman, 1989; Baxter & Kouparitsas, 2005). Bergman (2007) underlies that this puzzle could be partially solved regarding the effects that cycles magnitude could have on the above relationship.

Our approach is slightly different from the previous ones, since it basically relies on frequency-domain methods. By means of Singular Spectrum Analysis (SSA), Monte-Carlo SSA (MC-SSA), Multi-Taper Method (MTM), Maximum Entropy Method (MEM) and Multi-channel SSA (M-SSA) we analyze the three selected series both in the univariate and multivariate contexts. These tools allow us to extract the dominant periodic and quasi-periodic cycles characterizing each time series, thus gaining deeper insights into the eventually common behavior of GDP fluctuations across the three nations. In particular, they allow to

precisely isolate the oscillations accounting for most of the series variability, i.e. the leading cyclical components of these economic systems.

Spectral analysis methods are widely spread in digital signal processing, oceanography, meteorology, and so on, but their application in socio-economic fields has been generally very limited due to the low performance of classical spectral estimation techniques applied to economic time series¹ (Granger & Hatanaka, 1964; Granger, 1966; Granger, 1969). However, new developments in spectral estimation algorithms help to curb spectral leakage, thus allowing fruitful applications of spectral analysis in various economic contexts (Lisi & Medio, 1997; Higo & Nakada, 1998; Atesoglu & Vilasuso, 1999; Baxter & King, 1999; A'Hearn & Woitek, 2001; Croux, Forni, & Reichlin, 2001; Aadland, 2005).

In our specific case, spectral analysis is applied to distinguish trends and higher frequency oscillations in the series, thus isolating and reconstructing periodic and quasi-periodic components actually accounting for most of the process total variance². What is more, the recent techniques adopted are specially well suited for short, noisy, and chaotic time series, without requiring particular properties of stationarity or ergodicity (Ghil, et al., 2002). This feature is particularly appreciable in economics, where most series are short and noisy³.

The rest of the paper is divided into 4 sections: an introduction to the methodology and the dataset explored; the univariate analysis of the three GDP series; a multivariate joint investigation, and some conclusions.

2. Dataset description and methodology

The analysis is based on Eurostat data from quarterly national accounts of Italy, the UK, and the Netherlands⁴. Unfortunately, the three series cover different time spans: the longest one is for the UK (53 y, 1955:01 – 2008:01, N=213), then the Netherlands (31 y, 1977:01 – 2007:04, N=124), and Italy (26 y, 1981:02 – 2007:01, N=104). We analyze seasonally-adjusted with working day correction GDP at market prices, measured in chain-linked volumes (constant prices, millions of euro, 2000 reference year).

As we can notice from figure 1 – and subsequently from figure 22 – the UK experiences the most fluctuations probably due to the more flexible structure of its labour market with respect to Italy and the Netherlands. In fact, the lower bargaining power of its trade unions allows more flexible adjustments of both wages and employment, thus determining more rapid responses both to positive and negative shocks, with more sudden recessions and more pronounced expansions. On the contrary, the high power of the

¹ In his seminal work Granger (1966) observes that economic variables possess a typical spectral shape: regardless of the time series length and the filtering specifications, their spectrum always shows a large bump at the zero frequency band, which is quickly but smoothly reabsorbed in the next bands. This is due to leakage phenomena in the classical spectral estimation of time series showing a large peak at some dominant frequency. In fact, some of the power associated with such frequency actually leaks into the estimate of the neighboring frequency bands. Most economic time series are affected by leakage as a consequence of the very high explanatory power of their trend.

² Recall that in ergodic stationary processes power spectrum can be interpreted as the distribution of the series variance with respect to each frequency (Gardiner, 1983).

³ Notwithstanding evidence of chaos in economic data has not been confirmed yet, Broomhead & King (1986) demonstrate that SSA works well even with mildly nonlinear processes, as economic series effectively prove to come from (Brock, 1986; Neftci & McNevin, 1986; Brock & Sayers, 1988; Frank & Stengos, 1988; Serletis, 1996; Stanca, 1999; Brock, 2000).

⁴ Quarterly national accounts are compiled in accordance with the European System of Accounts (ESA95). Data are available on the web at <http://epp.eurostat.ec.europa.eu/>.

Italian trade unions causes a more rigid reaction of the entire economic system⁵, while the Netherlands stays in the middle, with a labour market more close to the Scandinavian typology⁶.

Another issue highly influencing GDP fluctuations deals with the energetic resources: in fact, while the UK and the Netherlands have direct access to natural energetic factors such as gas, Italy is a strong oil net importer. Thus, energetic shocks such as those occurred in the 70s highly affect the Italian economic system with respect to the other two countries – e.g. the Netherlands is a gas net exporter –, with consequent impacts on GDP oscillations.

Finally, as we already noticed, the economic magnitude of these countries is quite dissimilar, with different relative importance and international relationships: the Netherlands is a small economy, consequently more open and more exposed to international shocks. On the contrary, Italy is more related to the Germany, while the UK to the US, with subsequent different effects on the respective macroeconomic aggregates.

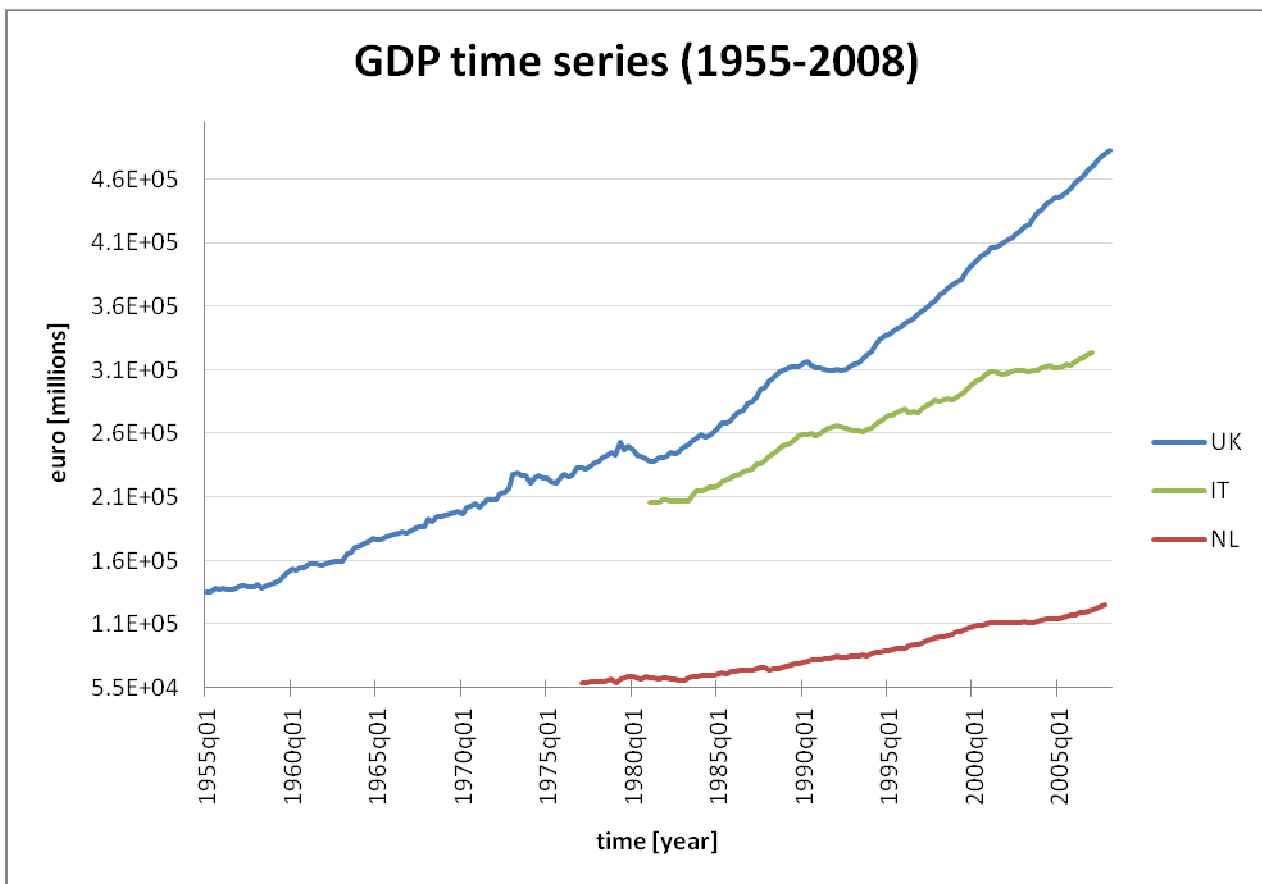


Figure 1 – GDP series for the UK, Italy, and the Netherlands: the different economic magnitudes and the respective trends-in-mean are clearly evident. Source: Eurostat.

⁵ For instance, the Italian Collective Agreements provide for very complex firing procedures in firms larger than 15 workers. As a consequence, medium- and large-size firms are discouraged to vary their employment level.

⁶ For the analysis of postindustrial economies and their typologies refer to Esping-Andersen (1999).

Univariate spectral analysis of GDP series permits both to extract and reconstruct their dominant periodicities, in order to inspect those fluctuations mainly characterizing the country-specific movements of this aggregate indicator of the national economic performance. Clearly, this kind of inquiry is purely phenomenological and does not require any *a priori* modeling, allowing a rigorous definition, quantification, and extraction of the long, medium, and short term components of the time series (Iacobucci, 2003). The lack of an underlying economic model in our empirical work could be somehow compensated by comparing our results with simulations based on NEDyM, the non-equilibrium dynamic model by Hallegatte, Ghil, Dumas, & Hourcade (2007), which predicts both endogenous business cycles of a few years in duration and near-annual periodicities. So far this model unfortunately provides a quite poor calibration, but the eventual agreement between empirical results and the model stylized facts could perhaps afford a validation.

Concerning the methodology, the analysis is based on some recent spectral analysis tools⁷. First of all we apply SSA, which is a fully non-parametric data-adaptive method especially well suited for short, noisy, and chaotic time series (Vautard & Ghil, 1989; Vautard, Yiou, & Ghil, 1992). This linear technique provides an orthogonal decomposition of the time series lag-covariance matrix, i.e. a projection of its principal components onto the vector space of the series delay-coordinates. With respect to classical Fourier methods, SSA detects oscillations modulated in both amplitude and phase (Allen & Smith, 1996). Thus, the original signal is no more simply decomposed into periodic sine and cosine functions, but rather into data adaptive waves possibly exhibiting non-constant amplitude and/or phase. This clearly represents a notable advantage when dealing with complex system dynamics, as in Economics.

In a preliminary step we apply SSA to extract trends from each raw series, then re-performing the analysis on the correspondent detrended profiles. This use of SSA as a data-adaptive detrending tool allows us to focus on the dominant cyclical fluctuations nonetheless neglecting the marked trend-in-mean behaviour of the GDP series (Granger & Hatanaka, 1964). Then, in order to identify periodic or quasi-periodic activity in the signal, Vautard & Ghil (1989) suggest to adopt the eigenvalue spectrum⁸, since it has been demonstrated that the *near-equality* of eigenvalue pairs *in phase quadrature* may be associated with oscillatory patterns⁹ (Ghil & Mo, 1991). Once the eigenvalue pairs are detected and their associated components reconstructed in order to inspect their time-domain characteristics, we identify their dominant frequency by means of MEM, a parametric technique giving an high-resolution estimate of the signal power spectrum by a stepwise extrapolation of the corresponding autocorrelation function (Jaynes, 1982). In fact, since each signal is characterized by its dominant frequencies, MEM is particularly helpful in identifying the main spectral peaks defining the oscillatory movements reconstructed through SSA decomposition.

Then, the significance of the reconstructed signal is tested against a data-adaptive red background noise through MC-SSA. This technique implements the surrogate data method (Theiler, Eubank, Longtin,

⁷ The authors are currently developing a methodological appendix in order to help the non-specialist reader to understand the basic characteristics of each spectral method and their reciprocal linkages. We apologize for eventually unclear methodological aspects and direct for the moment the interested readers to a methodological survey directly addressed to economists (Sella, 2008b).

⁸ It is a plot of the time series eigenvalues ranked by order, with the correspondent confidence bars. It is a fundamental tool, since the lag-covariance matrix decomposition in SSA is defined by the solution of the associated eigenvalue problem, where the corresponding eigenvectors provide the orthonormal basis for the signal decomposition.

⁹ Intuitively, each eigenvalue represents the fraction of total variance explained by the associated component: if two components explain more or less the same variance and their modes are in phase quadrature, they may represent an oscillatory pair. Thus, with some additional techniques we can identify both slow modes and either regular or irregular oscillations from a background noise and/or other uninteresting processes. In our empirical analysis the two features above are detected by the Same Frequency and the Strong FFT criteria implemented in the SSA-MTM toolkit, the open-source software we adopted (<http://www.atmos.ucla.edu/tcd/ssa/>).

Galdrikian, & Farmer, 1992) by creating different realizations of the null hypothesis (NH) noise in order to compute the respective surrogate confidence intervals: whenever an oscillatory pair falls outside the correspondent surrogate bar the NH is rejected at the chosen confidence level, since that way the test suggests that an AR(1) process is not a good description for the behaviour of the associated oscillation (Ghil & Vautard, 1991; Vautard, Yiou, & Ghil, 1992; Allen & Smith, 1996). Finally, the white and the locally white¹⁰ background noise NH are tested by MTM, a non-parametric technique generally applied to time series characterized by both broadband and line spectral components, i.e. respectively continuous parts and pure sinusoids (Thomson, 1982; Mann & Lees, 1996).

3. Univariate analysis

The following section presents univariate country-specific results. Notwithstanding the different time span of the series partially limits the comparisons among these countries, we can find some cyclical regularities across the sample. This could be an evidence of some common features in the dynamics of industrialized economies (cf. e.g. Blanchard & Watson, 1987; Stanca, 1999; Stock & Watson, 2002; Stock & Watson, 2005). This intuition is further strengthened by the remarkable correspondence among our multi-country empirical findings and the theoretical predictions in NEDyM.

3.1. Italy (1981:2 – 2007:1)

This time series covers 26 y, consisting of 104 quarterly observations. The preliminary SSA reveals a quite particular shape of the eigenspectrum¹¹ (fig. 2): no clear noise background is identifiable, while the first two components can be reasonably recognized as a trend. In fact, they jointly explain the 92% of the series total variance and both satisfy the *Kendall nonparametric test for global trend identification*¹² (Kendall & Stuart, 1968). Moreover, a MEM analysis on the joint SSA reconstruction of components 1 and 2 shows an associated dominant periodicity of about 46 y (fig. 3), which candidates the reconstructed signal as the probable trend-in-mean of the Italian GDP series¹³.

¹⁰ The locally white is a colored noise process varying slowly but arbitrarily with frequency (Mann & Lees, 1996). It allows testing against noise backgrounds with complicated structures.

¹¹ The results for Italy are computed with filtering window length $M=34$, but are robust to changes in its size. This feature is fundamental in determining the resolution adopted, since the larger M , the higher the periodicity we can look for in the analysis; however, an excessively high M makes the autocovariance function dominated by statistical error (Vautard & Ghil, 1989; Vautard, Yiou, & Ghil, 1992).

¹² The procedure is implemented in the Do Trend Test of the SSA-MTM toolkit.

¹³ Note that the MEM spectrum shows the typical spectral shape of economic variables described by Granger (1966). As the author underlines, this recurring shape is mostly due to the pervasive presence of trends. As a consequence, once the trend is removed the eigenspectrum shows a more usual shape (see fig. 6). In addition notice that our analysis is unfortunately limited by the very short length of our time series. This fact could involve a not clear distinction between trend and ultra low-frequency components, the first being characterized by a systematic linear or nonlinear behavior changing over time and not repeating at all, while the second ones by a formally similar structure which repeats itself in systematic intervals over time however not captured within the time range of our data.

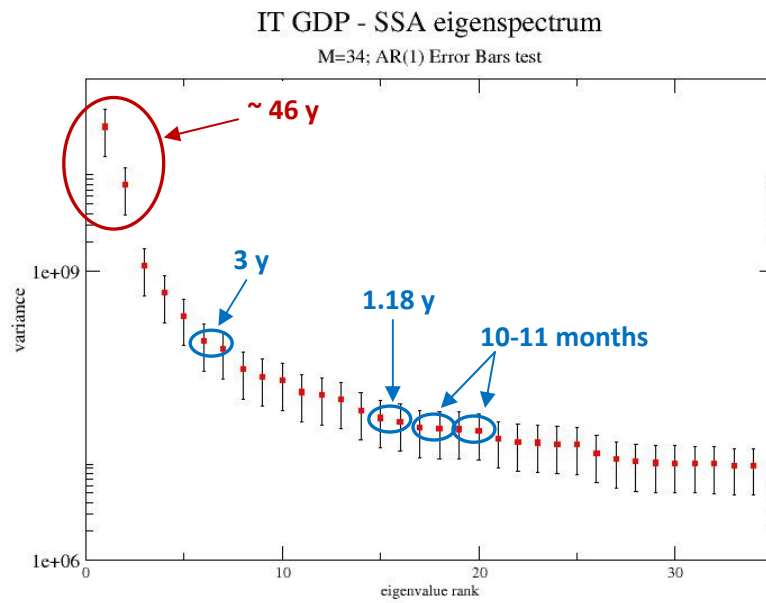


Figure 2 – Italian GDP series: SSA eigenspectrum. Trend components are circled in red, while potential SSA pairs in light blue; the error bars show an ad hoc range of the estimation errors based on the estimated decorrelation time of the series. The pairs are selected considering the results of both the Same Frequency and the Strong FFT tests (see notes 9 and 14), which identify potentially oscillatory patterns. On the contrary, the Do Trend Test recognizes those eigenvalues carrying in the series the most low-frequency and/or trend information. Since at this stage we essentially use SSA as a detrending low-pass filter, we linearly extract from the series the red-circled components and go on analyzing the detrended resultant reconstruction (fig. 5).

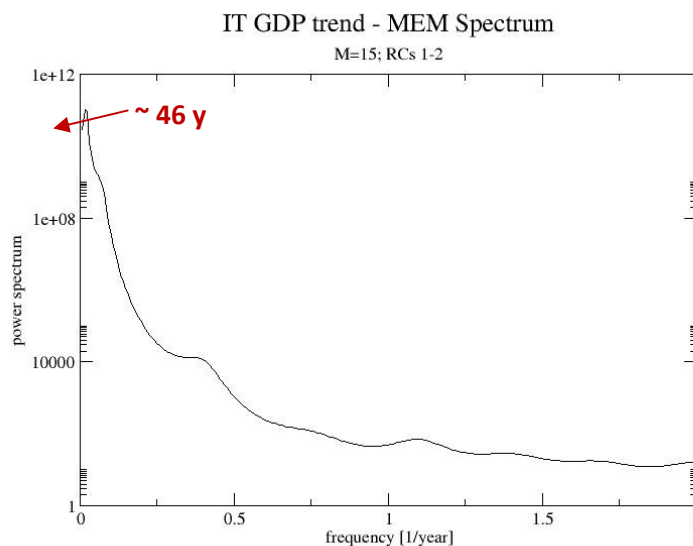


Figure 3 – Italian GDP: MEM spectrum of the joint reconstruction of SSA components 1 and 2, i.e. the most explicative trend components. We show this plot since it clearly points out the typical spectral shape of economic variables, which are generally highly affected by memory (Granger, 1966). Since this power spectrum is not much informative, we mainly concentrate our subsequent analysis on the detrended series and their associated cyclical movements.

Looking at the time-domain reconstruction of the trend (fig. 4), we notice three main divergences between the raw and reconstructed signals, corresponding respectively to the early 80s, the late 80s – early 90s, and the year 2000. This evidence is in accordance with the results of a combined time-/frequency-domain analysis on the Italian GDP growth rate breaks for the same time span (Sella, 2008). These structural breaks are due both to some peculiarities of the recent Italian economic history, and to some international events: first of all, both the 1973 and the 1979 strong energetic shocks could have lengthen the current recession phase, while the reprise starting around 1985 could have been stretched by both the dollar depreciation and the energetic counter-shock, characterized by the reduction of the oil price: it particularly affected the Italian economy due to its high dependence on energetic factors.

Moreover, the deviation from trend occurred in the period 1990-95 is probably due to the restrictive policy adopted by the government in order to recover both the high debt/GDP ratio, and the high inflation due to the dissolute deficit-spending policy of the 80s. In fact, the Maastricht Treaty fixed some economic policy targets very hard for the current financial situation of Italy. Finally, the deviation around year 2000 could be related the worldwide crisis of the stock exchange and the New Economy. Unfortunately, the analyzed series is quite short and we suffer of finite-length sampling error; thus, the interpretation of breaks at the beginning of the series must be particularly careful.

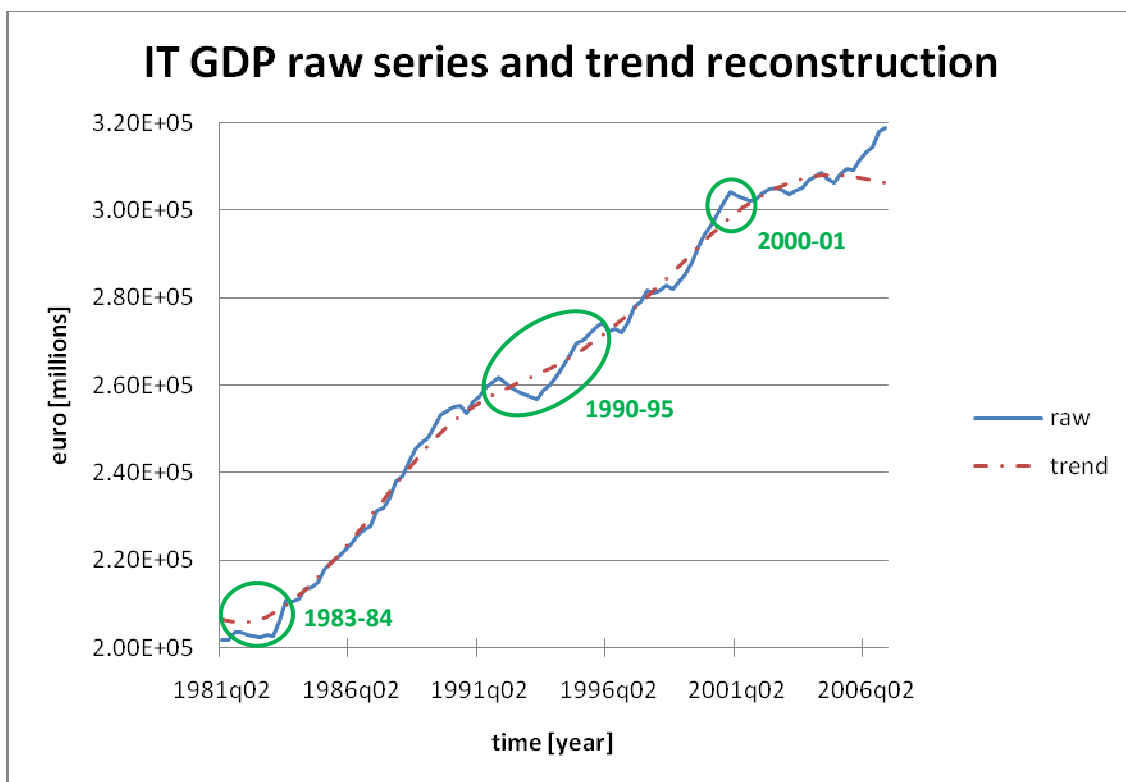


Figure 4 – Italian GDP: SSA reconstruction of components 1 and 2 against the raw series. This graph is particularly helpful in identifying the purely cyclical behavior of GDP, since the difference between the raw series and the reconstructed trend effectively rules out most of the variation eventually due to breaks in the trend. The plot shows three areas where the two series mainly differ, corresponding respectively to the early 80s, the late 80s – early 90s, and the period 2000-01. All these points actually correspond to critical periods for the Italian economy.

Going on with the spectral decomposition, the SSA pairs 6-7, 15-16, 17-18, and 19-20 (light blue circles in fig. 2) potentially represent quasi-periodic oscillations¹⁴, but their explicative power is very low (less than 1% of the total variance). Nevertheless, MEM analyses on the reconstructed signals reveal periodicities of about 3 y, 1.18 y, and 10-11 months. The above results sensibly change when the series is detrended¹⁵ (fig. 5) in order to focalize on purely cyclical components.

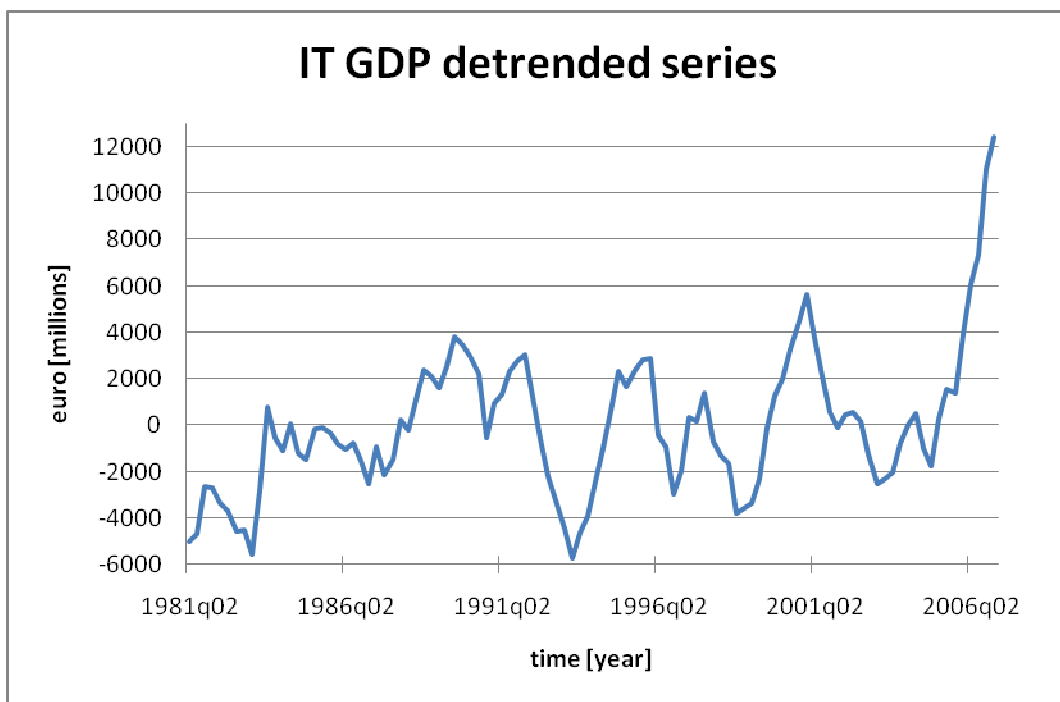


Figure 5 – Detrended Italian GDP: here SSA is applied as detrending tool in order to rule out from the raw series those components accounting for most of its variability (components 1 and 2), since GDP is a process highly affected by memory. The most of our subsequent analysis effectively concentrates just on the detrended series, since they allow to focus our attention simply on purely cyclical behavior.

The new eigenvalue spectrum ($M=34$) shows a more standard shape: the first six components clearly differ from the other ones, which essentially represent the background noise (fig. 6). The most explicative eigenvalue clusters satisfying both the Same Frequency and the Strong FFT criteria are represented by pairs 1-2, 5-6, and 7-8 (light blue circles in fig. 6), but Monte Carlo analysis identifies as significantly different from a data-adaptive red background noise at the 90% confidence level just the first two pairs (fig. 7). They are associated to oscillatory movements of about 6 y and 3 y periodicities, reconstructed in the time domain in fig. 8. In its lower panel it is noticeable the lengthening of the recession phase in the early 80s, due to the real effects of the 1979 oil shock. Moreover, both the medium- and the short-size oscillations show a descending movement in the very last part of the sample, probably due to both the German stagnation phase in 2006-07 and the restrictive financial act the Italian government had to pass in order to cover the previous balance difficulties.

¹⁴ In fact, they satisfy both the Strong FFT and the Same Frequency criteria, which respectively test whether the EOFs associated with the potential clusters have the same dominant frequency, and which portion of the total variance they account for at such frequency.

¹⁵ SSA is an useful detrending tool since it allows to isolate and reconstruct those components presumably determining the series trend. Technically, the detrended series is obtained by linearly subtracting from the raw series those components accounting for the trend behavior.

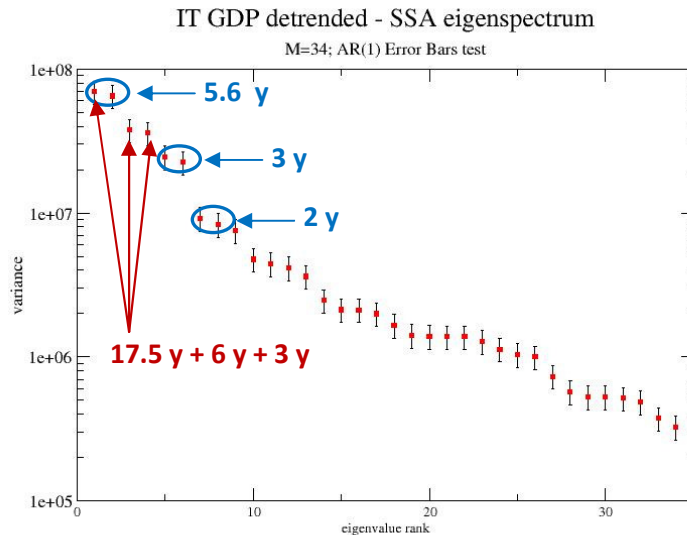


Figure 6 – Detrended Italian GDP: SSA eigenspectrum. The light blue circles highlight the most explicative SSA pairs which are nearly in phase quadrature, thus suspected to represent nonlinear quasi-harmonic oscillations in the data. Some trend and/or low-frequency residual information is contained in the components pointed out by the red arrows. However, since the shape of the spectrum is now quite standard and this residual information could be an artificial by-product of finite length, we do not perform any further detrending.

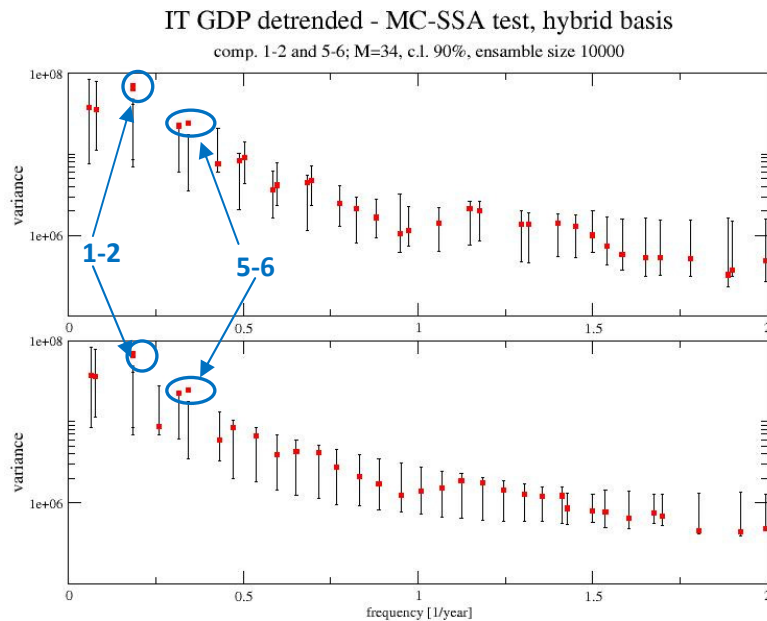


Figure 7 – Detrended Italian GDP: hybrid basis MC-SSA test (SSA pairs 1-2 and 5-6; $M=34$). The basis has been determined successively adding one by one those components staying above their error bar. This represents for each EOF the 90% variance found in the state-space direction defined by that EOF in an ensemble of 10000 red noise realizations (Ghil, et al., 2002). The upper panel tests against the EOFs directly derived from the data covariance matrix, while the lower panel upon the EOFs computed from the expected covariance matrix of the NH process. This double testing against both the data and the NH EOFs is justified since these last ones should avoid the artificial variance compression inherited from SSA, therefore lowering the first-type-error probability (Allen & Smith, 1996). The above plot shows that our data fit well to an AR(1) process when components 1-2 and 5-6 are included into the NH.

In addition, the Do Trend Test suggests the presence of some residual trend and/or low-frequency movements in the 1st, 3rd, and 4th eigenvalues (red arrows in fig. 6), which jointly account for the 44% of the detrended series total variance. The MEM spectrum of their joint reconstruction shows three main peaks,

corresponding respectively to periodicities of 17.5 y, 6 y, and 3 y. However, since we are analyzing a detrended series, the Do Trend Test probably points out some significant low-frequency components¹⁶.

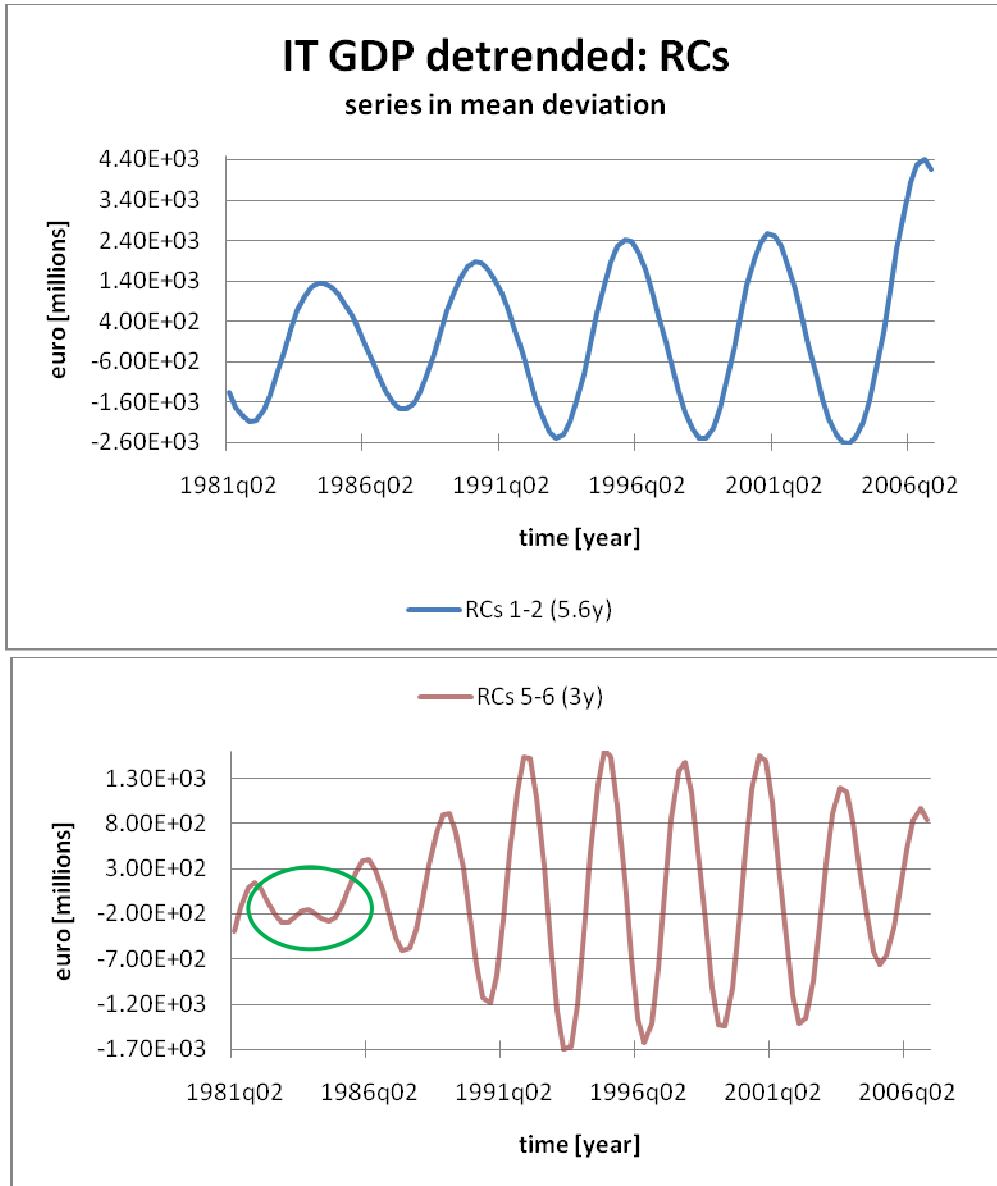


Figure 8 – Detrended Italian GDP: time-domain reconstruction of SSA pairs 1-2 and 5-6. They respectively represent dominant quasi-periodic oscillations of 5.6 y and 3 y periods, which significantly differ from a red noise background at the 90% confidence level. In the lower panel we can notice the lengthening of the recession phase in the early 80s (green circle) due to the real effects of the energetic shock. Time-domain reconstructions are provided by both SSA and MTM, thus allowing a visual inspection of each detected periodicity.

Summing up, the SSA of the detrended Italian GDP series singles out a low-frequency fluctuation of 17.5 y period and two oscillations of about 6 y and 3 y, which significantly differ from a red noise background null

¹⁶ The fact that SSA component 1 satisfies both the Do Trend Test and the pairing up criteria is quite puzzling: a MEM analysis on the joint reconstruction of components 3 and 4 points out periodicities of about 13 y and 3 y. Thus, when adding component 1 to the reconstruction, the estimated power spectrum shows in addition the 6 y peak, which is more or less recovered in the oscillatory pair 1-2, while the low frequency peak is now localized at a lower value. A possible interpretation is that components 1 and 2 have the same dominant frequency, but while component 2 has an almost periodic behaviour with a nearly constant envelope, component 1 shows a non-constant envelope, which affects the series behaviour at low frequencies.

hypothesis. In order to strengthen these findings, we test the series against different NH, and distinguish between harmonic and narrowband components (Mann & Lees, 1996).

We perform an MTM analysis on the GDP detrended series with 3 tapers, resolution 2 y^{-1} , and median smoothing window width 0.4. Red and locally white noise NH point out more or less the same significant narrowband components. The MTM power spectra in fig. 9 show significant peaks corresponding to oscillations of about 4.4 y, 1.4 y and 8-10 months. On the contrary, the white noise NH reveals significant peaks at 13 y and about 3 y periodicities. Finally, concerning the purely harmonic analysis, the Variance-Ratio Test highlights dominant frequencies corresponding to periodicities of 5.5 y (99%), 3 y (90%), 1.5 y, 1.2 y, and 7-9 months (95%).

These dissimilar results depending on the hypothesized background noise are justified by the fact that red noise is an AR(1) process concentrating the most rejection capability at low frequencies, while white noise is a flat AR(0) with an higher threshold at high frequencies than the corresponding red noise. Thus, depending on the background noise we suppose to perturb our system, we obtain different statistically significant oscillations. This point raises a particularly important issue, since some economic literature identifies in the AR(1) process a good description for the GDP behaviour (cf. Hess & Iwata, 1997; McConnell & Perez-Quiros, 2000). Thus, in our case a red noise NH would in some way mean to identify most of the economic signal as noise, disregarding the low-frequency components presumably related to the series trend and/or low-frequency components. On the contrary, such high-frequency periodicities are generally filtered away when testing against the white noise null hypothesis, which mostly identifies low-frequency components (Sella, 2008).

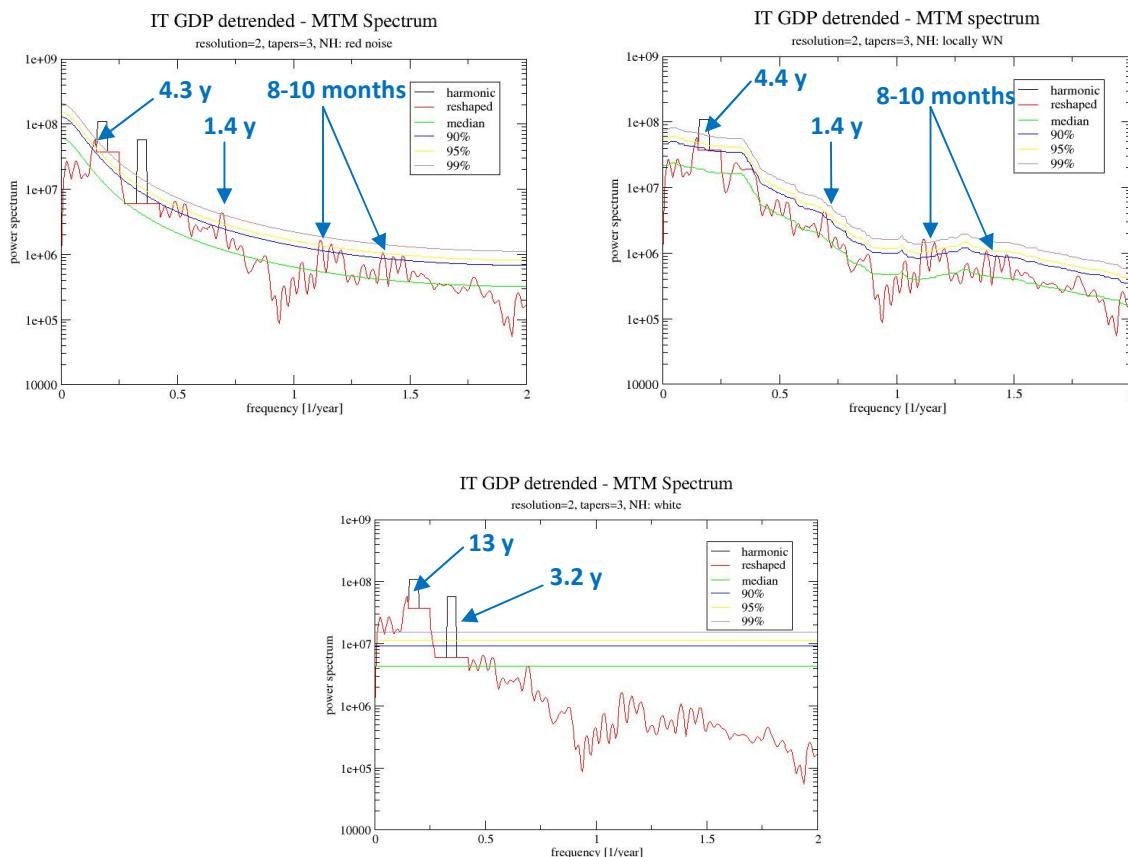


Figure 9 – Detrended Italian GDP: MTM spectra tested against red (high left panel), locally white (high right panel), and white (low panel) background noise NH. The first two tests show similar results, highlighting periodicities of about 4.4 y and some near annual ones; on the contrary, the third test identifies significant lower-frequency components. The legend of the three plots allows to recognize the 90%, 95%, and 99% confidence levels.

Summarizing all the above results, a) the 3 y periodicity is detected in both the raw and the detrended series by almost all methods applied; b) an about 5-6 y periodicity is recurrent in the detrended series and it proves significantly different from a red noise background process; c) some near annual periodicities emerge¹⁷. These features are in line with NEDyM simulations (Hallegatte, Ghil, Dumas, & Hourcade, 2007).

3.2. The Netherlands (1977:1 – 2007:4)

This analysis closely follows the precedent scheme. However, since the variability of the raw series is highly affected by its characteristic trend-in-mean, we mainly analyze the detrended series to focalize on purely cyclical components.

This second time series is a bit longer than the previous one, covering 31 y (N=144). As for the Italian case, the preliminary SSA reveals a non-standard eigenspectrum (fig. 10). The analyses are performed with $M=41$, but the results are robust to changes in the embedding dimension. Again, the first two components can be recognized as a trend, satisfying the Do Trend Test and jointly explaining about the 94% of the total variance. Together with components 3 and 5, the associated periodicities of the corresponding time-domain reconstructions are 59 y and 5.6 y.

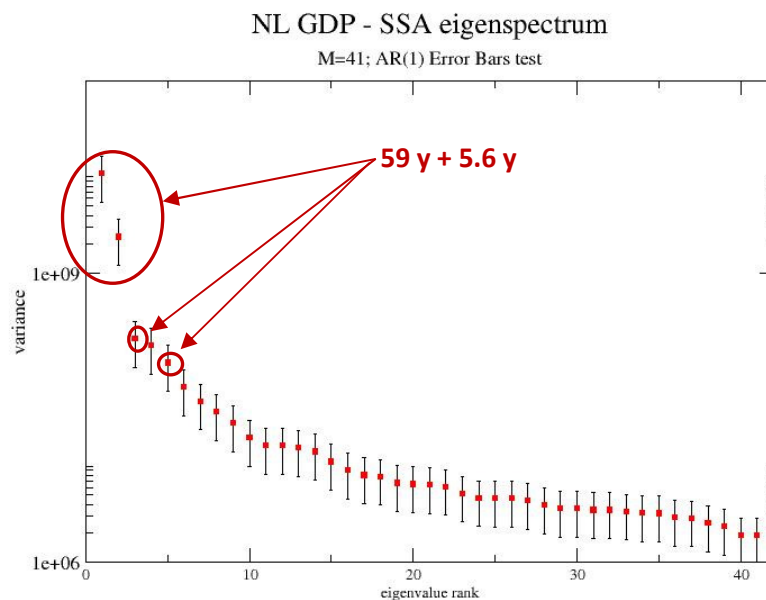


Figure 10 – NL GDP series: SSA eigenspectrum. The components satisfying the Do Trend Test (1, 2, 3, 5) are circled in red. They jointly explain about 96% of the series total variance and the associated periodicities are 59 y and 5.6 y. As in fig. 2 the eigenspectrum error bars refer to the estimation error.

The trend reconstruction (fig. 11) follows quite well the raw data, except for the period 2000-06. This quite lasting discordance is probably due to the characteristic openness of the Dutch economy with respect to international shocks: in fact, in 2000 and 2001 the US experienced two consistent crises which had many international effects. Moreover, the plot does not show any critical episode during the oil shock and later, since the Netherlands is a gas net exporter and is relatively unaffected by oil crises.

¹⁷ Note that all these oscillations are detected in MTM when testing for harmonic components.

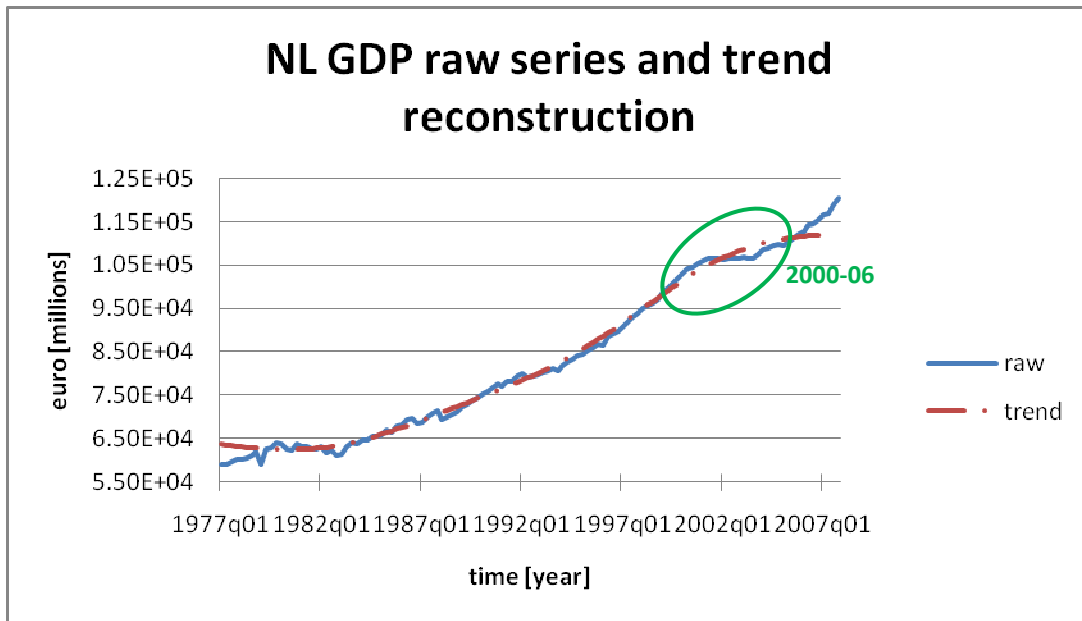


Figure 11 – NL GDP: SSA reconstruction of components 1 and 2 against the raw series. The reconstruction fits quite well except for the period 2000-06. As usual, its extremes differ from the raw series due to finite sampling.

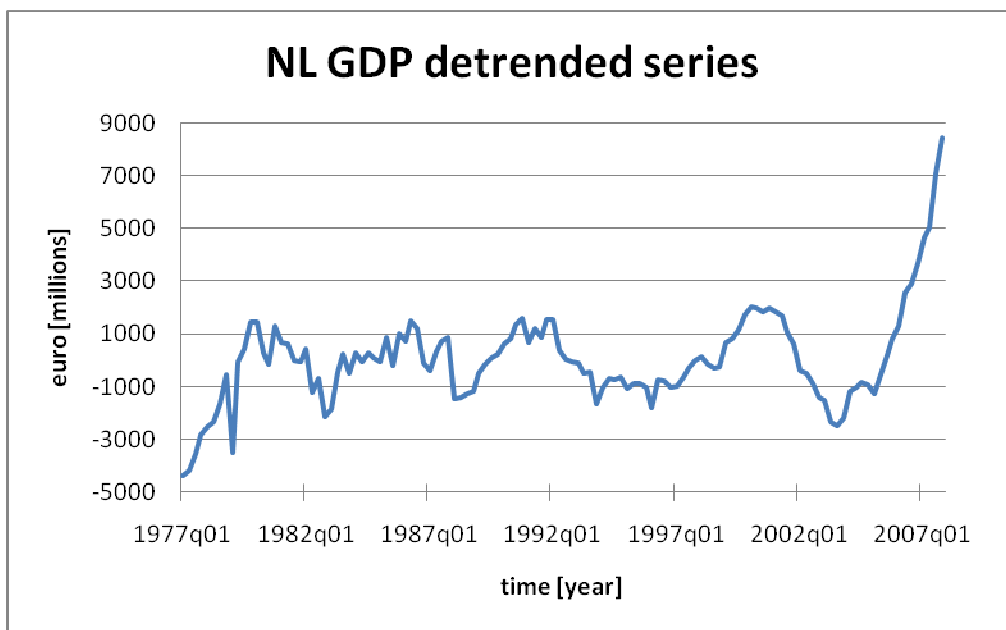


Figure 12 – Detrended NL GDP: SSA is applied as detrending tool in order to rule out from the raw series those components accounting for most of the total variability (here components 1 and 2).

Due to the strong trend-in-mean behavior characterizing GDP, we analyze the detrended series (fig. 12) to focalize on its dominant cyclical components. The first five eigenelements of this eigenvalue spectrum ($M=41$, fig. 13) clearly move away from the rest: they probably identify the series statistical dimension¹⁸, but some important cycles are possibly hidden in the noise tail. In fact, the MC-SSA test identifies at the 90% c.l. the eigenpairs 1-2 and 19-20 (fig. 14), respectively associated to a 7.7 y and an 8 months

¹⁸ It represents the upper bound for the minimal number of degrees of freedom required to describe the attractor associated to the data generating process (Vautard & Ghil, 1989), i.e. the rank of the first eigenvalue clearly distinct from the noise tail in the eigenspectrum plot.

oscillations. On the contrary, the about 3 y and the 1.7 y periodicities, respectively due to the eigenpairs 6-7 and 10-11, are not significantly different from a data-adaptive red noise process.

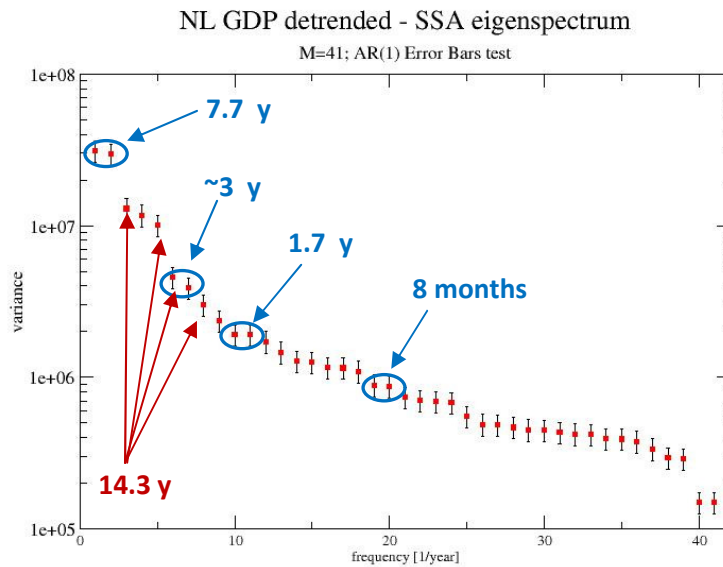


Figure 13 – Detrended NL GDP: SSA eigenspectrum. The light blue circles show the most explicative SSA pairs, while the red arrows point out the components detected by the Do Trend Test, which are associated to a low-frequency oscillation. As in the previous analysis, residual trend and ultra-low frequency information is not ruled out for the second time since the eigenspectrum shape is quite standard, with a few eigenvalues clearly emerging from a quite flat noise tail.

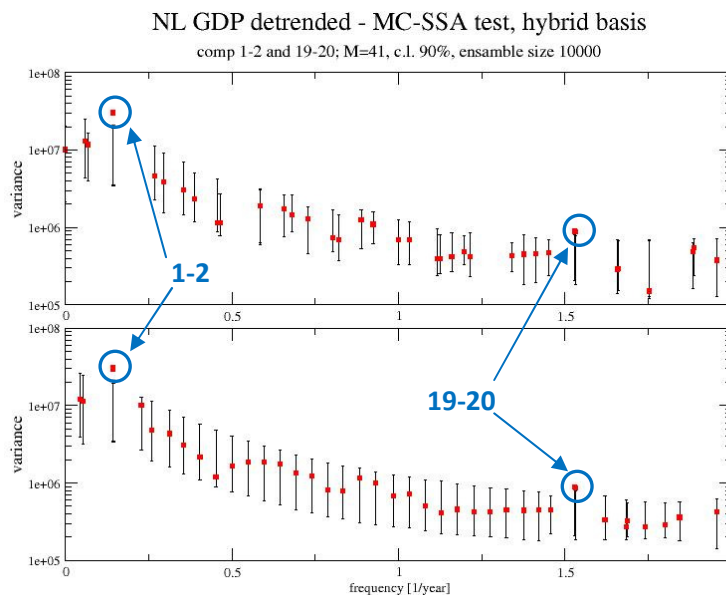


Figure 14 – Detrended NL GDP: hybrid basis MC-SSA test (SSA pairs 1-2 and 19-20; $M=41$). In this case the oscillations significantly different from a red noise process correspond to periods of 7.7 y and 8 months.

Analogously to the Italian case, fig. 15 shows the 3 y cycle; however, in this case its significance is rejected by MC-SSA, and also by MTM. Nonetheless, this periodicity is a common feature of all three countries analyzed.

More in details, MTM (3 tapers, resolution 2 y^{-1} , median smoothing window width 0.4) detects a 6-7 y cycle (fig. 15) clearly distinct at the 99% c.l. from all three background noise NH (red, white, and locally white). On the contrary, the circa annual periodicities are singled out just by the red and locally white benchmarks¹⁹. Finally, both the 5 y and the near annual periodicities are detected by the harmonic analysis.

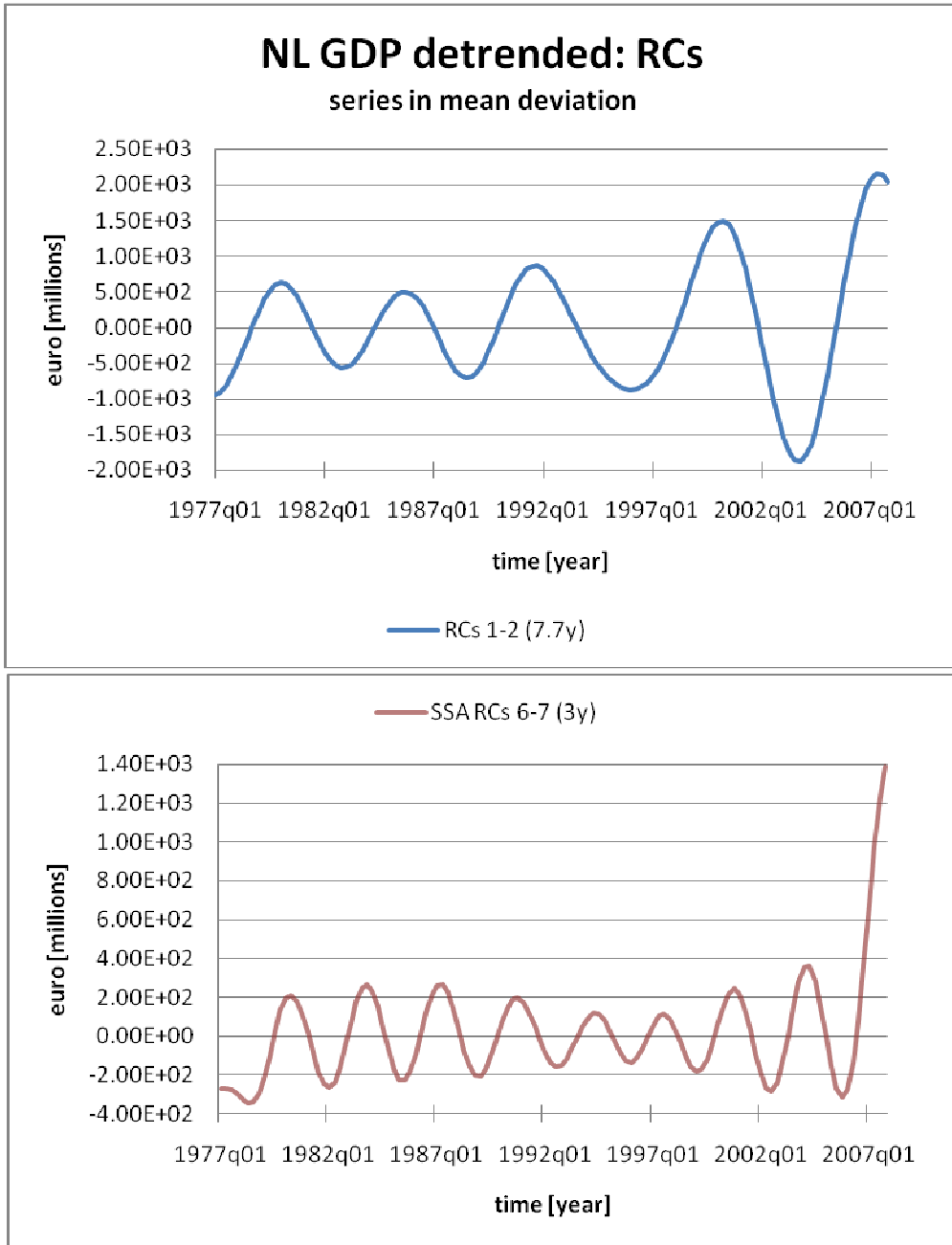


Figure 15 – Detrended NL GDP: reconstruction of SSA pairs 1-2 (7.7y) and 6-7 (3y). In the upper panel we use $M=38$ instead of $M=41$ in order to obtain a more clear reconstruction. This was possible since the main characteristics of the analysis proved robust to changes in the filtering window. Notice that the upper plot shows a slightly increasing variance starting from around year 2000, which has been previously detected as a sort of break in the trend reconstruction of fig. 11.

¹⁹ The omitted plots are freely available from the authors.

Summarizing the results for the Netherlands, a) the 3 y periodicity is detected just by the SSA of the detrended series as an oscillation not significantly different from the red noise null (90% c.l.); b) an about 6-7 y cycle recurs both in the raw and the detrended series, proving significantly different from red, white, and locally white noise background process; c) some near annual periodicities emerge both in SSA and MTM analyses. These features are highly in accordance with the previous findings concerning the Italian economy.

3.3. The UK (1955:1 – 2008:1)

This is the longer time series we dispose of, covering more than 53 y (N=213): this fact allows us to widen our analysis, including some insight on the possible effects many important international occurrences had on the economic systems. In addition, this quite strong economy is out of Euroland, allowing us to make some comparisons between Euro- and non-Euro-area countries.

The preliminary SSA ($M=70$) reveals that the first three components can be recognized as a trend and jointly explain the 93% of the total variance, with an associated periodicity of about 100 y (fig. 16).

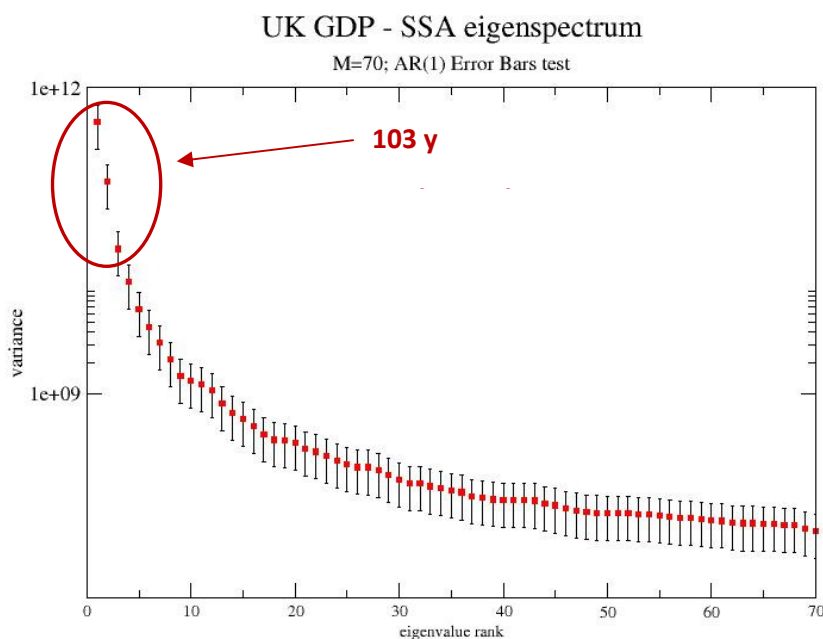


Figure 16 – UK GDP series: SSA eigenspectrum. The components satisfying the Do Trend Test (1, 2, 3, 4, 5, 6) jointly account for an about 100 y period oscillation (see reconstruction in fig. 17).

Fig. 17 highlights two main intervals where the raw series differs from the trend reconstruction: they are respectively 1979-84, the period following the second energetic shock, and 1990-95, a phase highly determinant for the evolution of EMU, when the UK had to sustain the costs of the Iraq first war.

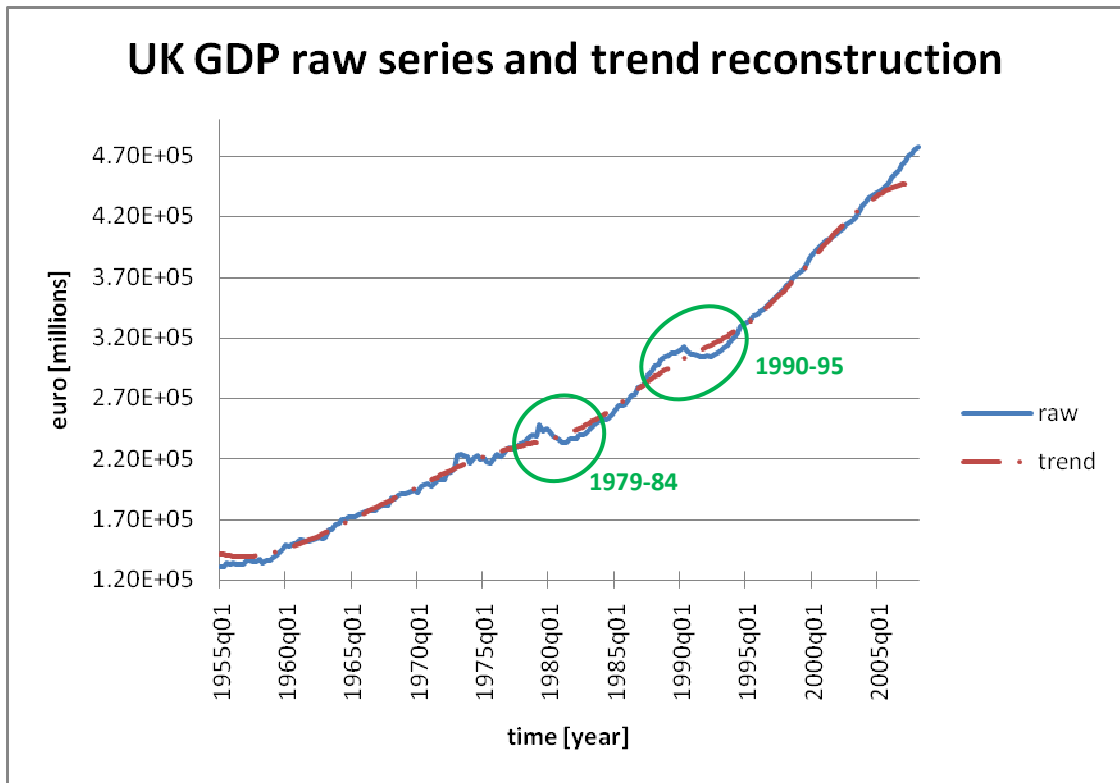


Figure 17 – UK GDP: SSA reconstruction of the trend components 1, 2, and 3. In 1979-84 and 1990-95 the raw series reliably differs from the trend. The generally strange behavior of the trend reconstruction at both the beginning and the end of the sample period is mostly due to finite sampling. Clearly this feature is transferred to the detrended series: notice e.g. the steep performance of the series in the last sample period of fig. 18.

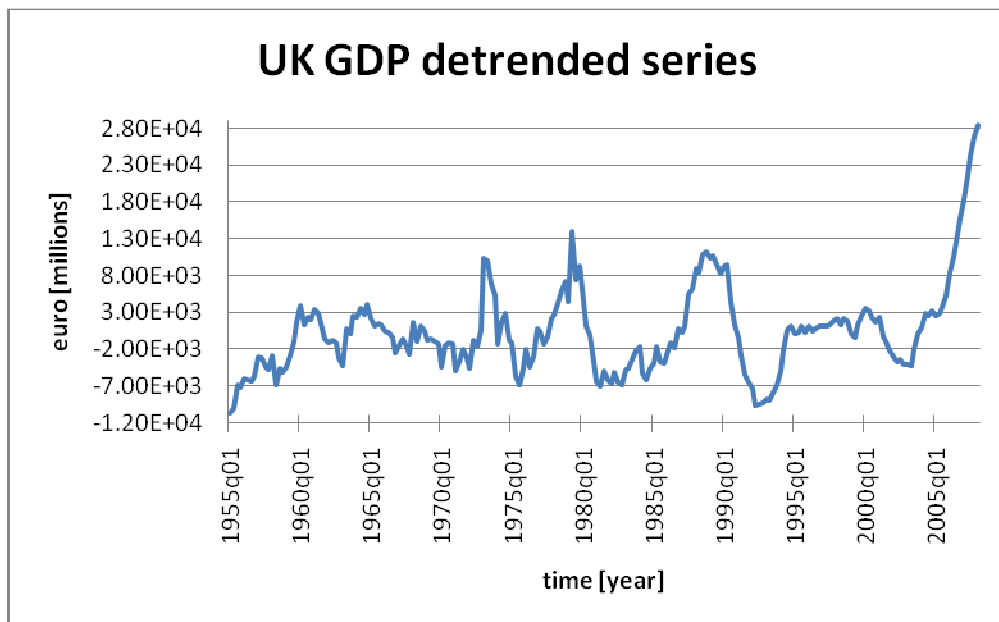


Figure 18 – Detrended UK GDP: the series is computed by linearly subtracting from the raw series the first three SSA components, since SSA acts as a low-pass filter.

After detrending, the SSA eigenspectrum ($M=70$) shows a clear noise tail starting from the 14th component (fig. 19). The MC-SSA singles out at the 90% c.l. the eigenpair 1-2 and the component 3, which respectively

account for a 10 y periodicity and an about 5 y movement (fig. 20). The other detected oscillatory pairs, including those accounting for the 3 y periodicity, are not statistically significant when tested against the data-adaptive red noise.

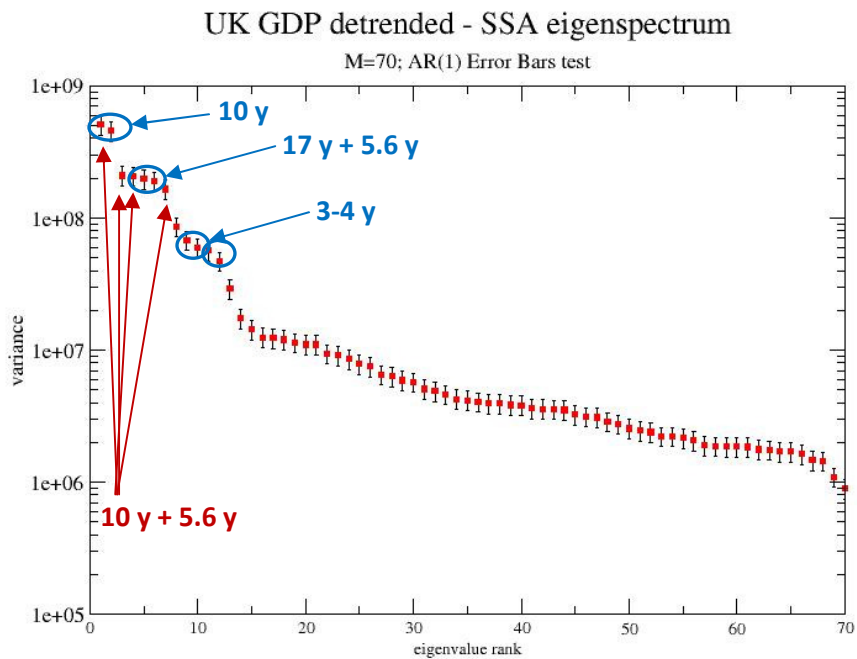


Figure 19 – Detrended UK GDP: SSA eigenspectrum. As above, the light blue circles show the most explicative SSA pairs, while the red arrows point out the low-frequency components detected by the Do Trend Test.

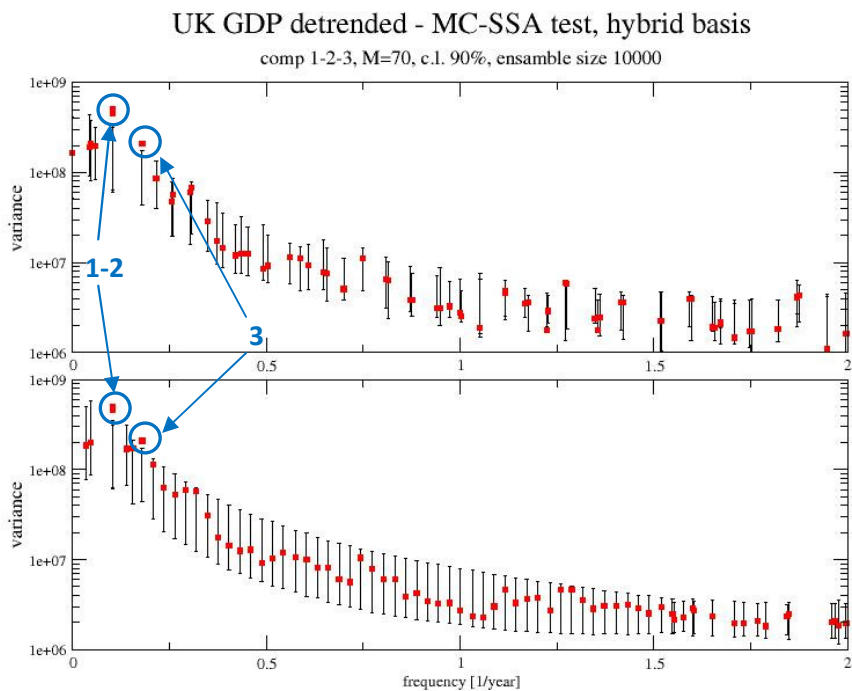


Figure 20 – Detrended UK GDP: hybrid basis MC-SSA test (SSA eigenelements 1-2 and 3; $M=70$). The oscillations significantly different from the data-adaptive red noise NH correspond to 10 y and 5 y movements. Notwithstanding in the upper panel the pair 28-29 falls out of the corresponding surrogate bars, in the lower panel it is well represented by the expected red noise process. Thus, for this pair we cannot reject the NH.

However, the situation changes in MTM (tapers, resolution $2 y^{-1}$, median smoothing window width 0.4), where the 3 y cycle is always significant at least at the 90% c.l., both when testing against the different noise benchmarks, and when considering purely harmonic signals. Moreover, as in the previous cases, the near annual periodicities are singled out by both the red and the locally white noise null, while the white one detects an additional 6 y periodicity. The 3 y and 6 y cycles are reconstructed in fig. 21.

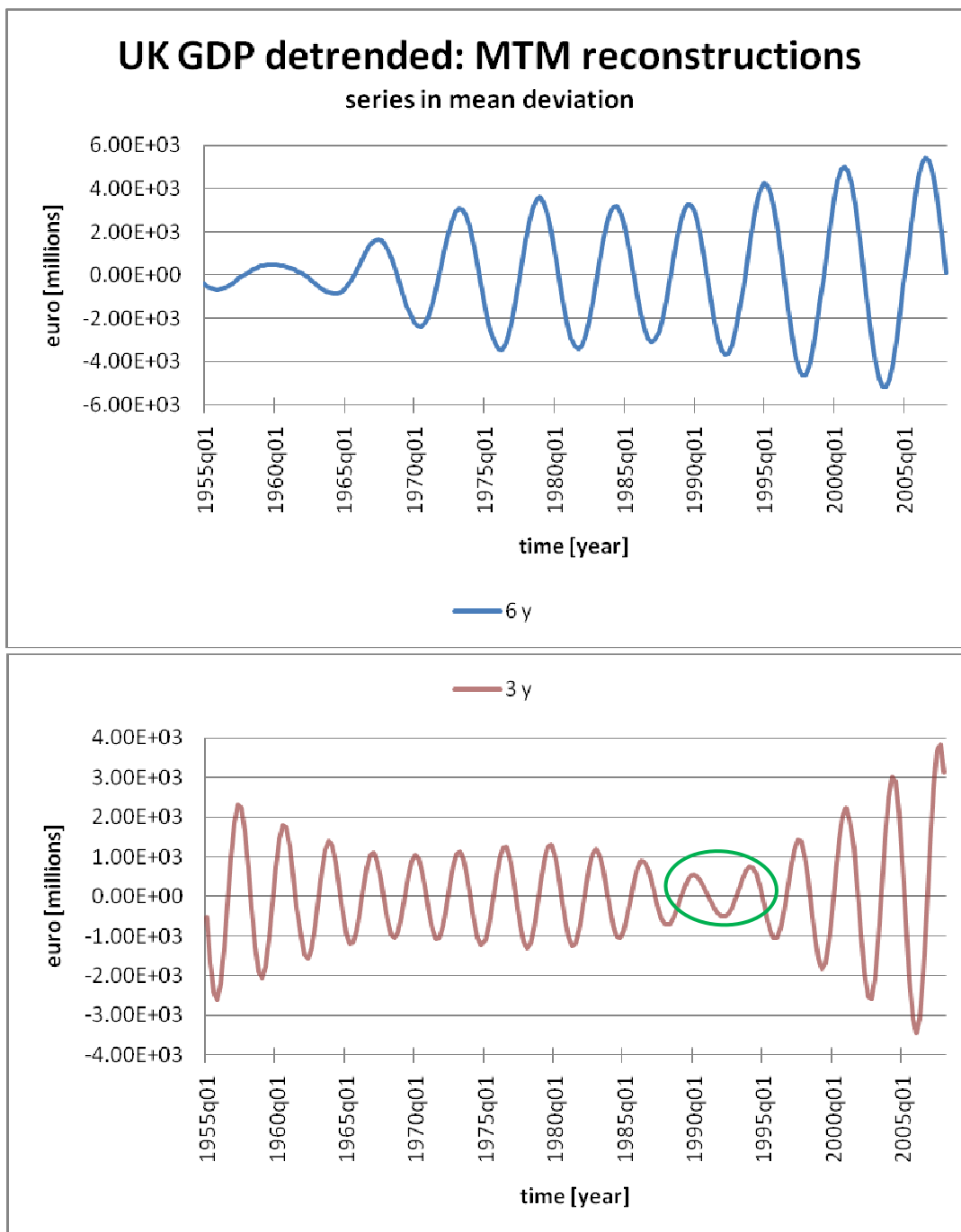


Figure 21 – Detrended UK GDP: reconstruction of the 6 y and 3 y periodicities using MEM. These cycles are significantly different from a white noise process respectively at the 99% and 90% c.l.. Notice that the lower panel shows a slightly different behavior of the envelope around the years 1990-95, i.e. the same time span we already observed in fig. 17 where the raw series perceptibly comes far away from the reconstructed trend.

Summarizing the results for the UK, a) the 3 y periodicity is detected by all methods, but it is significantly different from noise just in MTM²⁰; b) an about 6 y cycle is singled out by both SSA and white-noise MTM, but does not pass the red noise test in both the methods; c) some near annual periodicities emerge in MTM when testing against both the red and the locally white NH.

3.4. Brief comparison of the three analyses

From the above results we can notice a quite homogeneous cyclical behaviour among Italy, the UK, and the Netherlands. Notwithstanding the different length of these series and the peculiarities of each economy, three types of oscillations always emerge (table 1):

- a “medium” business cycle fluctuation²¹ of around 5-6 y, which is a little longer in period for the Netherlands and a little shorter for Italy;
- a “short” business cycle of about 3 y, which is not always significant when tested against the red noise null;
- some near annual periodicities (both a little longer and a bit shorter than a precise annual cycle), usually detected by MTM but generally incorporated into the SSA noise tail.

However, notice that sometimes the across-methods results do not show a complete and full agreement, particularly concerning significance. This could be mainly due to the different algorithms characterizing each method and its tests. However, as we explained the main periodicities are quite systematically detected in all series.

GDP detrended series			
Method	IT	UK	NL
SSA	17.5y*, ~6y, 3y, 2y*	10y, 17y* + 5.6y, 3-4y*	14.3y*, 7.7y, 3.3y*, 1.7y*, 8 months
MTM red	4.3y, 1.4y, 8-10 months	8y, 3.3y, 1.3y, 6-11 months	6.7y, 1.6y, 1.16y, 6-8 months
MTM white	13y, 3.2y	12.5y, 5.9y, 3.3y	7.1y
MTM locally white	4.3y, 1.4y, 8-10 months	10.6y, 2.9y, ~1.3y, 6-11 months	6.25y, 6-12 months
MTM harmonics	5.5y, 3y, ~1.3y, 6-9 months	3y, 2.17y, ~1.5y, 8-12 months	16.7, 5y, ~1.3y, 6-10 months

Table 1 – GDP detrended: summary of main results of the univariate analysis for Italy, the UK, and the Netherlands. (*) stands for not significantly different from the red noise null hypothesis at the 90% c.i. in MC-SSA. Notably, the results are quite homogeneous across countries.

These results are in line with those obtained by Hallegatte, Ghil, Dumas, & Hourcade (2007) within their NEDyM framework. This is a non-equilibrium dynamic model introducing investment dynamics and non-equilibrium effects into a Solow growth model, hence focusing on the economic consequences of both institutional and technological inertia²². From simulations, NEDyM exhibits both endogenous business

²⁰ This discrepancy between MC-SSA and MTM tests could be due to the different way each algorithm specifies the noise benchmark. In fact, notice that in MC-SSA of fig. 20 the SSA eigenpair 9-10 associated with the about 3 y periodicity is very close to the upper extreme of the surrogate confidence interval.

²¹ Recall that in literature there is no full agreement about the effective extension of economic business cycles, but it is quite common to consider them as the result of an about 2-8 (or 10) years data generating process (Crivellini, Gallegati, Gallegati, & Palestrini, 2007).

²² The importance of these factors in economic dynamic models have been particularly emphasized by Invernizzi & Medio (1990).

cycles of a few years in duration and some near-annual fluctuations. In particular, a 5-6 y periodicity emerges in profits, production, and employment, consistently with the mean business cycles period (Zarnowitz, 1985; King & Watson, 1996; Kontolemis, 1997). Moreover, a subannual oscillation affects wages, employment, and consequently production, probably due to labour-market dynamics locked to the one-year period due to seasonal forcing. However, further coupled theoretical-empirical research is necessary on such issue, together with the extension of the empirical analysis to other macroeconomic time series (e.g. investment, consumption, wages, employment, import, export, etc.). In fact, so far NEDyM calibration is quite poor, but further agreements among empirical and simulated results could provide a substantial validation.

Finally, in order to strengthen our empirical findings we perform in the next section a simple multivariate analysis by Multichannel SSA (M-SSA), which allows to observe the joint dynamics of our series.

4. Multivariate analysis: M-SSA

The SSA method provides in addition a multivariate approach through M-SSA (Broomhead & King, 1986b; Ghil, et al., 2002). This allows to analyze time series vectors or maps, in order to study the spatiotemporal structures characterizing the solution of systems of partial differential equations and the properties of their reconstructed attractor (Temam, 1997).

In our case, M-SSA allows the joint analysis of the main periodicities characterizing all three countries under study. We apply it to the centered detrended series of GDP over the common time span of the three series²³ (1981:01-2006:04, 26 y, N=104; fig. 22). As we can notice, the UK detrended GDP is the most oscillatory series, as the factors explained in section 2 suggest. This behavior is more evident in the detrended series than in the growth rates (omitted plot), since the detrended ones are not affected by the variability due to breaks in the trend, while the growth rates are not able to separate the respective contributions.

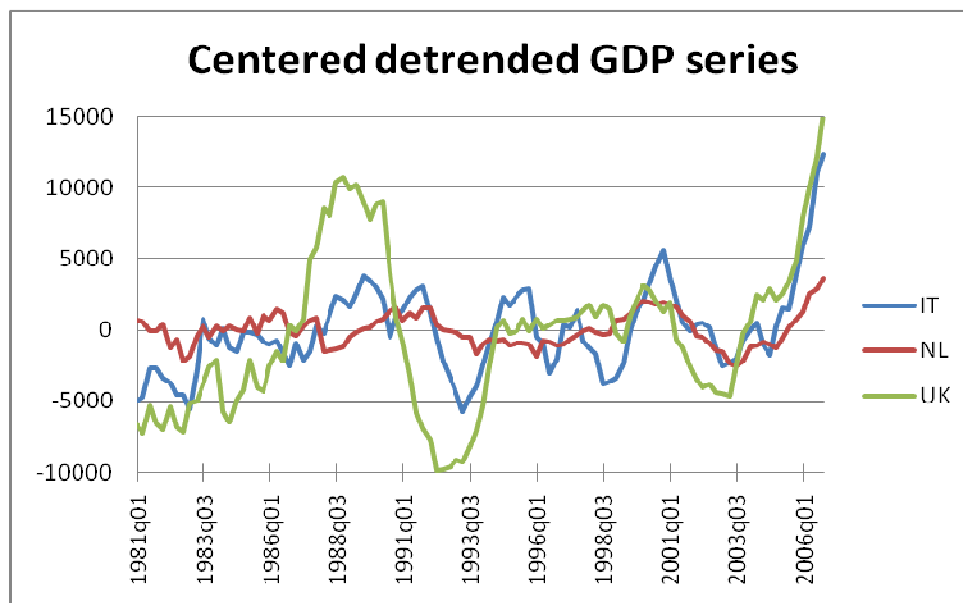


Figure 22 – Plot of the centered detrended GDP for Italy, the Netherlands, and the UK for the time span 1981:01-2006:04. It is quite evident the different behavior of the series for the UK on one side, and the Euro-area countries on the other. This fact is confirmed by the subsequent analysis (fig. 24).

²³ The dataset is obtained through a standard preliminary prefiltering by Principal Components Analysis. In addition, we use the “Reduced” algorithm to compute the data covariance matrix, since it proves more efficient.

The components potentially coupled in the eigenspectrum are the 1-2, 3-4, and 5-6: the first two pairs pass the 90% c.l. Chi-Squared significance test, together with component 6 (fig. 23). They correspond respectively to a 9 y, a 6 y and a 3 y cycles which prove significantly different from a red noise background process. As we can notice from table 1, these results are highly in line with the univariate analysis findings, strengthening the evidence of a common business cycle behaviour among the three countries. Furthermore, this joint analysis shows that both the 3 y and 6 y oscillations significantly differ from a red noise null hypotheses process eventually perturbing the system, thus overcoming the significance problems occurred in the univariate case.

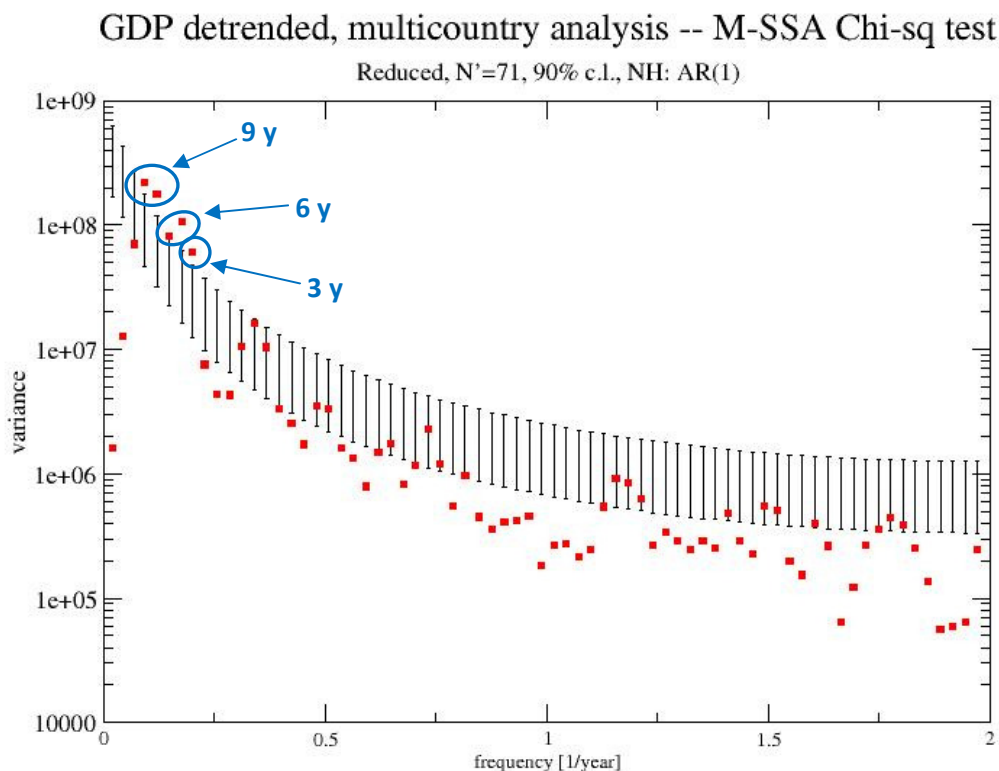


Figure 23 – M-SSA Chi-Squared test: the components falling outside the error bars are significantly different from a red noise null hypothesis at the 90% c.l.. Note that an high number of eigenvalues are outside the AR(1) error bars from below: we neglect them since the portion of the variance they explain is smaller than the benchmark process we are interested in.

However, the joint M-SSA reconstructions of the 6 y and 3 y cycles (fig. 24) show some differences in their phasing: the 6 y periodicity highlights a slight lead-lag relation between Italy and the Netherlands, while the UK shows an opposite phase, which becomes more evident when looking at the 3 y reconstruction. On the contrary, in this second case the two continental countries are completely in phase, showing in addition very similar amplitudes.

From an economic point of view, these differences are justifiable looking both at the respective international interdependencies (the UK is highly influenced by the US, while Italy and the Netherlands are more linked to Germany and France), and at the peculiar features of each economy: while the UK is more reactive to both external and internal variations, entering the recession phase earlier and recovering more quickly, the two continental countries show lagged and more synchronized reactions.

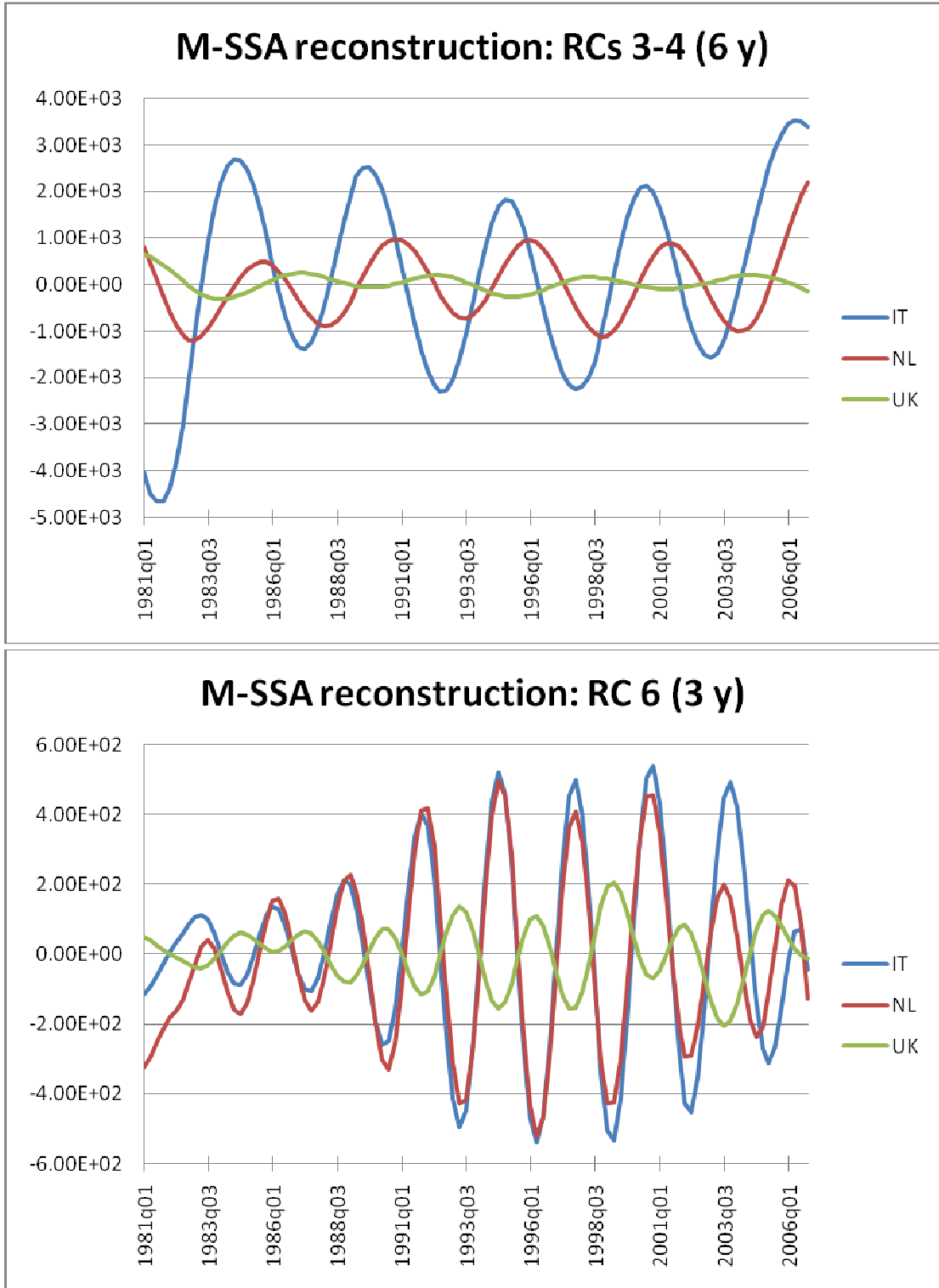


Figure 24 – M-SSA reconstructions: the 6 y and 3 y periodicities of each GDP series are computed and compared.

5. Conclusions

Both the univariate analyses of each individual time series and the multivariate analysis of the complete data set suggest that the economic systems of Italy, The Netherlands, and the United Kingdom are characterized by very similar cyclical behavior. The similarity in cycle lengths, however, does not necessarily

translate into synchronization among the respective aggregates. In particular, the two continental countries show highly synchronized fluctuations, while the United Kingdom seems completely in phase opposition. Thus, our advanced spectral analyses seem to indicate a higher degree of economic integration over Euroland, even though this point requires a more detailed investigation. Indeed, our whole data set shows a high degree of synchronization between Italy and The Netherlands, starting well before 1992; this has to be weighed against the fact that, while the announcement effects of the EMU began immediately after the Maastricht Treaty, in 1992, the genuine effects started even later, namely in 1999. Bergman (2007), though, suggests that the linkages among European countries date back to the immediate post-war period, when these countries started planning several efforts to integrate their national markets.

To conclude, further work is needed to sharpen the multivariate analysis and to apply the entire methodology across a broader and more heterogeneous sample of economies, including also non-European ones, as well as to additional macroeconomic variables. Moreover, from a theoretical point of view, a deeper understanding of the similarities between the real economies and NEDyM results is needed as well.

References

- Aadland, D. (2005). Detrending time-aggregated data. *Economic Letters*, 89, 287-93.
- A'Hearn, B., & Woitek, U. (2001). More international evidence on the historical properties of business cycles. *Journal of Monetary Economics*, 47, 321-46.
- Allen, M. R., & Smith, L. A. (1996). Monte Carlo SSA: detecting irregular oscillations in the presence of colored noise. *Journal of Climate*, 9, 3373-404.
- Arnold, L. G. (2002). *Business Cycle Theory*. New York, U.S.A.: Oxford University Press.
- Arthur, W. B., Durlauf, S. N., & Lane, D. A. (1997). *The Economy as an Evolving Complex System II: Proceedings*. MA, USA: Addison-Wesley Publishing Company.
- Atesoglu, H. S., & Vilasuso, J. (1999). A band spectral analysis of exports and economic growth in the United States. *Review of International Economics*, 7 (1), 140-52.
- Baxter, M., & King, R. G. (1999). Measuring business cycles: approximate band-pass filters for economic time series. *Review of Economics and Statistics*, 81, 575-93.
- Baxter, M., & Kouparitsas, M. A. (2005). Determinants of business cycle movement: a robust analysis. *Journal of Monetary Economics*, 52, 113-57.
- Baxter, M., & Stockman, A. C. (1989). Business cycles and the exchange-rate regime. *Journal of Monetary Economics*, 23, 377-400.
- Bergman, M. U. (2007). How similar are European business cycles? In G. L. Mazzi, & S. G. (eds), *Growth and Cycle in the Eurozone*. Basingstoke, Hampshire, UK: Palgrave MacMillan.
- Blanchard, O. J., & Watson, M. W. (1987). Are business cycles all alike? *NBER Working Papers Series* (1392).
- Brock, W. A. (1986). Distinguish random and deterministic systems: abridged version. *Journal of Economic Theory*, 40, 168-95.
- Brock, W. A. (2000). Whither nonlinear? *Journal of Economic Dynamics and Control*, 24, 663-78.

- Brock, W. A., & Sayers, C. L. (1988). Is the business cycle characterized by deterministic chaos? *Journal of Monetary Economics*, 22, 71-90.
- Broomhead, D. S., & King, G. (1986). Extracting qualitative dynamics from experimental data. *Physica D*, 20, 217-36.
- Broomhead, D. S., & King, G. P. (1986b). On the qualitative analysis of experimental dynamical systems. In S. Sarkar (Ed.), *Nonlinear Phenomena and Chaos* (pp. 113-44). Bristol, England: Adam Hilger.
- Chiarella, C., Flaschel, P., & Franke, R. (2005). *Foundations for a Disequilibrium Theory of the Business Cycle*. Cambridge, Great Britain: Cambridge University Press.
- Coe, D., & Helpman, E. (1995). International R&D spillovers. *European Economic Review*, 39, 859-87.
- Crivellini, A., Gallegati, M., Gallegati, M., & Palestrini, A. (2007). Output fluctuations in G7 countries: a time-scale decomposition analysis. In G. L. Mazzi, & G. Savio (Eds.), *Growth and Cycle in the Eurozone* (pp. 45-59). Basingstoke, Hampshire, UK: Palgrave MacMillan.
- Croux, C., Forni, M., & Reichlin, L. (2001). A measure of comovement for economic variables: theory and empirics. *Review of Economics and Statistics*, 83 (2), 232-41.
- De Haan, J., Inklaar, R., & Sleijpen, O. (2002). Have business cycles become more synchronized? *Journal of Common Market Studies*, 40, 23-42.
- Dickerson, A. P., Gibson, H. D., & Tsakalotos, E. (1998). Business cycle correspondence in the European Union. *Empirica*, 25, 51-77.
- Esping-Andersen, G. (1999). *Social Foundations of Postindustrial Economies*. Oxford, Great Britain: Oxford University Press.
- Fatás, A. (1997). EMU: Countries or regions? Lessons from the EMS experience. *European Economic Review*, 41, 743-51.
- Frank, M. Z., & Stengos, T. (1988). Some evidence concerning macroeconomic chaos. *Journal of Monetary Economics*, 22, 423-38.
- Frankel, J. A., & Rose, A. K. (1998). The endogeneity of the optimum currency area criteria. *Economic Journal*, 25, 1009-25.
- Frisch, R. (1933). *Propagation Problems and Impulse Problems in Dynamic Economics. Economic Essay in Honor of Gustav Cassel*. London: George Allen and Unwin.
- Gardiner, C. W. (1983). *Handbook of Stochastic Methods. For Physics, Chemistry and the Natural Sciences*. Berlin Heidelberg: Springer-Verlag.
- Ghil, M., & Mo, K. (1991). Interseasonal oscillations in the global atmosphere. Part I: Northern Hemisphere and Tropics. *Journal of the Atmospheric Sciences*, 48 (5), 752-79.
- Ghil, M., & Vautard, R. (1991). Interdecadal oscillations and the warming trend in global temperature time series. *Nature* (350), 324-27.
- Ghil, M., Allen, M. R., Dettinger, M. R., Ide, K., Kondrashov, D., Mann, M. E., et al. (2002). Advanced Spectral Methods for Climatic Time Series. *Review of Geophysics*, 40 (1), 1-41.
- Goodwin, R. M. (1967). A Growth Cycle. In C. Feinstein (Ed.), *Socialism, Capitalism, and the Economic Growth*. Cambridge, Cambridge University Press, Great Britain.

- Granger, C. W. (1969). Investigating causal relations by econometric models and cross-spectral methods. *Econometrica* , 37 (3), 424-38.
- Granger, C. W. (1966). The typical spectral shape of an economic variable. *Econometrica* , 34 (1), 150-61.
- Granger, C. W. (1966). The Typical Spectral Shape of an Economic Variable. *Econometrica* , 34 (1), 150-61.
- Granger, C. W., & Hatanaka, M. (1964). *Spectral Analysis of Economic Time Series*. Princeton, New Jersey: Princeton University Press.
- Hallegatte, S., Ghil, M., Dumas, P., & Hourcade, J.-C. (2007). Business Cycles, Bifurcations and Chaos in Neo-Classical Model with Investment Dynamics. *Journal of Economic Behaviour and Organization* .
- Hess, G. D., & Iwata, S. (1997). Measuring and Comparing Business-Cycle Features. *Journal of Business and Economic Statistics* , 15 (4), 432-44.
- Hicks, J. (1950). *A Contribution to the Theory of the Trade Cycle*. Oxford, Great Britain: Oxford University Press.
- Higo, M., & Nakada, S. K. (1998, December). How can we extract a fundamental trend from an economic time-series? *Monetary and Economic Studies* .
- Iacobucci, A. (2003). Spectral analysis for economic time series. *OFCE Working Papers* (2003-07).
- Inklaar, R., & De Haan, J. (2001). Is there really a European business cycle? A comment. *Oxford Economic Review* , 53, 215-20.
- Invernizzi, S., & Medio, A. (1990). On lags and chaos in economic dynamic models. *Journal of Mathematical Economics* , 20, 521-50.
- Jaynes, E. T. (1982). On the rationale of Maximum-Entropy Methods. *Proceedings of the IEEE* , 70 (7), 939-52.
- Juglar, C. (1862). *Des Crises Commerciales et de Leur Retour Periodique en France, en Angleterre et aux Etats-Unis*. Paris, France: Guillaumin.
- Kaldor, N. (1940). A Model of the Trade Cycle. *Economic Journal* , 50, 78-92.
- Kalemli-Ozcan, S., Sørensen, B. E., & Yosha, O. (2001). Economic integration, industrial specialization, and the asymmetry of macroeconomic fluctuations. *Journal of International Economics* , 55, 107-37.
- Kendall, M. G., & Stuart, A. (1968). *The Advanced Theory of Statistics*. London, Great Britain: Griffin.
- King, R. G., & Rebelo, S. T. (2000). Resuscitating Real Business Cycles. *NBER Working Paper Series* (7534).
- King, R., & Watson, M. (1996). Money, prices, interest rates and the business cycles. *Review of Economics and Statistics* , 78, 35-53.
- Kontolemis, Z. (1997). Does growth vary over the business cycle? Some evidence from the G7 countries. *Economica* , 64, 441-60.
- Krugman, P. (1993). Lessons of Massachussets for EMU. In F. Giavazzi, & F. (. Torres, *The Transition to Economic and Monetary Union in Europe*. New York: Cambridge University Press.
- Kydland, F. E., & Prescott, E. C. (1982). Time to Build and Aggregate Fluctuations. *Econometrica* , 50 (6), 1345-70.

- Lisi, F., & Medio, A. (1997). Is a random walk the best exchange rate predictor? *International Journal of Forecasting*, 13, 255-67.
- Long, J. B., & Plosser, C. I. (1983). Real Business Cycles. *Journal of Political Economy*, 91 (1), 39-69.
- Mann, M. E., & Lees, J. M. (1996). Robust Estimation of Background Noise and Signal Detection in Climatic Time Series. *Climatic Change*, 33, 409-45.
- McConnell, M. M., & Perez-Quiros, G. (2000). Output Fluctuations in the United States: What Has Changed Since the Early 1980's? *American Economic Review*, 90 (5), 1464-76.
- Neftci, S. N., & McNevin, B. (1986). Some evidence on the non-linearity of economic time series: 1890-1981. *Working Paper of the C.V. Starr Center for Applied Economics* (86/26).
- Rose, A. K., & Engel, C. (2002). Currency unions and international integration. *Journal of Money, Credit, and Banking*, 34, 1067-89.
- Sella, L. (2008). *Economic Fluctuations Analysis: Alternative Approaches*. Turin: unpublished Ph.D. Thesis.
- Serletis, A. (1996). Is there chaos in economic time series? *Canadian Journal of Economics*, 29 Special Issue, S210-12.
- Slutsky, E. (1927). *The Summation of Random Causes as a Source of Cyclic Processes, III(1)*. Conjuncture Institute, Moscow.
- Stanca, L. M. (1999). Are business cycles all alike? Evidence from long-run international data. *Applied Economics Letters*, 6, 765-69.
- Stanca, L. M. (1999). Asymmetries and nonlinearities in Italian macroeconomic fluctuations. *Applied Economics* (31), 483-91.
- Stanca, L. M. (2001). La teoria delle fluttuazioni economiche: una prospettiva storica. *Working Paper, Dip. Economia Politica, Univ. Milano Bicocca* (34).
- Stock, J. H., & Watson, M. W. (2002). Has the business cycle changed and why? *NBER Macroeconomic Annual*.
- Stock, J. H., & Watson, M. W. (2005). Understanding changes in international business cycle dynamics. *Journal of the European Economic Association*, 3 (5), 966-1006.
- Temam, R. (1997). *Infinite-Dimensional Dynamical Systems in Mechanics and Physics* (2nd ed.). New York, USA: Springer-Verlag.
- Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., & Farmer, J. D. (1992). Testing for nonlinearity in time series: the method of surrogate data. *Physica D*, 58, 77-94.
- Thomson, D. J. (1982). Spectrum estimation and harmonic analysis. *Proceedings of the IEEE*, 70, 1055-92.
- Vautard, R., & Ghil, M. (1989). Singular spectrum analysis in nonlinear dynamics, with applications to paleoclimatic time series. *Physica D*, 35, 395-424.
- Vautard, R., Yiou, P., & Ghil, M. (1992). Singular-Spectrum Analysis: a toolkit for short, noisy chaotic signals. *Physica D*, 58, 95-126.
- Zarnowitz, V. (1985). Recent Work on Business Cycles in Historical Perspective: a Review of Theories and Evidence. *Journal of Economic Literature*, 23 (2), 523-80.