Experimental evidence for the interplay between individual wealth and transaction network

Jie-Jun Tseng[1], Sai-Ping Li[1], and Sun-Chong Wang[2]

[1] Institute of Physics, Academia Sinica, Taipei 115 Taiwan
[2] Institute of Systems Biology and Bioinformatics, National Central University, Chungli 320 Taiwan

Abstract. We conduct a market experiment with human agents in order to explore the structure of transaction networks and to study the dynamics of wealth accumulation. The experiment takes place on the platform TAIPEX for 97 days with 2,095 effective participants and 16,936 times of transactions. From these data, the hybrid distribution (log-normal bulk and power-law tail) in the wealth is observed and we demonstrate that transaction networks should be scale-free and disassorative from time to time even for those with the size of the order of few hundred. We further discover that the individual wealth and its degree is correlated with a power-law growth which allows us to relate the exponent of the degree distribution to the Pareto index in the individual wealth.

PACS. 87.23.Ge Dynamics of social systems – 89.65.Gh Economics; econophysics, financial markets, business and management – 89.75.Da Systems obeying scaling laws – 89.75.-k Complex systems

1 Introduction

It has been widely observed that the distribution of wealth among individuals in various economies follows a remarkably simple pattern, namely, a power-law tail for the rich (also known as Pareto's law) and a log-normal distribution for the rest [1–4]. The wealth distribution $P_w(w)$ can thus be expressed as

$$P_w(w) \sim \begin{cases} w^{-(\alpha+1)} & \text{for } w \geq w_*, \\ \exp[-\frac{(\log w-\mu)^2}{2\sigma^2}] & \text{for } w < w_*, \end{cases} \tag{1}$$

where w_* indicates the threshold wealth for the transition between the log-normal and power-law distribution. α is a time-dependent parameter called *Pareto index* ranging from 1 to 2 for different countries, while μ and σ denote the mean and standard deviation of $\log w$, respectively.

Several attempts have been made to model the dynamics of wealth accumulation as an exchange process or the interactions among agents. [5–9]. Some of these models involve the concept of transaction networks while dealing with the interactions and recently the effects of the network topology on the wealth accumulation in these models have been explored [8, 10–13]. However, since it is difficult to study these networks empirically, the further knowledge regarding them is still unavailable. An alternative approach is proposed by Diego Garlaschelli and his colleagues, where they study the interplay between the World Trade Web (WTW) and the gross domestic product (GDP) of world countries [15–17]. Having all this in mind, we here propose yet another way to study the interplay between transaction networks and wealth accumulation, namely by analyzing the data from the market experiment on our platform.

In what follows, we first introduce our platform and the market experiment. We then report the findings for the topology of the transaction networks and discuss how it affects the wealth accumulation in our experiment. In the end, we summarize all the findings and propose its possible implication in the near future.

2 Experimental design and data set

The market experiment with anonymous volunteers from the Web took place on our platform from Dec. 2007 to Mar. 2008. The platform TAIPEX (Taiwan Political Exchange [1]), initiated in 2003, is a 24-hour Web-based prediction market which facilitates the trading of political futures contracts whose liquidation prices are coupled to specific election outcomes [21–23]. Through the internet, anyone with a Web browser can participate in the market experiment by an on-line registration. A private account with user-provided login name will be created for this registrant and the platform will deposit an initial amount of virtual money into the account immediately after successful registration. The information about user demography, price fluctuation and accumulated volumes are public to any browser irrespective of his registration or not, however, only registered user with a valid account can trade in the market. The minimal unit for the virtual money is set to one in our experiment and participants can buy

[1] http://socioecono.phys.sinica.edu.tw

bundles of futures contracts from the platform for a guaranteed price or buy the contracts from the market directly upon login. In our design, a given contract is associated with the liquidation price which equals to the percentage of votes that a candidate will get on the day of election. As for the bundle, it consists of contracts for all candidates in the race as well as for all the invalid casts. One should keep in mind that all the contracts in the account must be liquidated with the official results from the election at the end of experiment, therefore, the bundle price of 100 is fair since neither the user nor our platform loses.

Traders can place market or limit orders to either buy or sell futures contracts. These bid (ask) orders are then stored in the order book maintained by our platform which adopts the continuous double auction (CDA) as the order matching and price discovery mechanism. If no matches are found, market orders will expire immediately while the limit orders will stay in the book and wait for further matches with new orders. Nevertheless, these limit orders would either expire or be canceled by traders before the matches. Once the matches are found, the transaction is made immediately, followed by a corresponding balance within the trader's accounts. No fees would be charged upon the transaction or order submission. In addition to what we mentioned above, since the virtual money is used for the trading in our market, we run the experiment as a tournament to encourage participants for further trading. They can make investment decisions of their own free will to compete for the money prizes provided by us, however, only the one whose ultimate wealth ranks in top ten will be rewarded.

The data set comes from a market experiment during Dec. 2007 to Mar. 2008 on the 2008 Taiwan presidential election. In this experiment, we issued three futures contracts which consisted of two candidates from two major political parties in Taiwan (KMT and DPP) and one for any invalid ballots cast on the election day. Each account begins with no futures contracts but an amount of virtual money up to 10,000 units as the initial wealth. As we have mentioned earlier, a bundle consisting of three contracts is provided at price 100 by our platform. Afterward, the market prices of these three contracts should sum up to around 100 if the traders behave rationally or if the market is efficient. By the last day of experiment, we accumulate 16,936 entries of transactions from 39,209 entries of orders submitted by 2,095 effective participants and out of them, there are total 1,985 players who make successful transactions. Fig. 1 shows the price time series covering the experiment. The contracts for candidates from KMT and DPP are drawn in blue and green, respectively, while the contract for invalid ballots is drawn in gray. The intermittence of price spikes in the plot may originate in a multiplicative process with additive noise which is supposed to yield the power-law fluctuations [18,19] or the so-called stylized facts in the financial market [20]. In an earlier work [22], by demonstrating that these stylized facts can be reproduced in our experiment, we propose that our platform is a justified candidate for performing the market experiment.

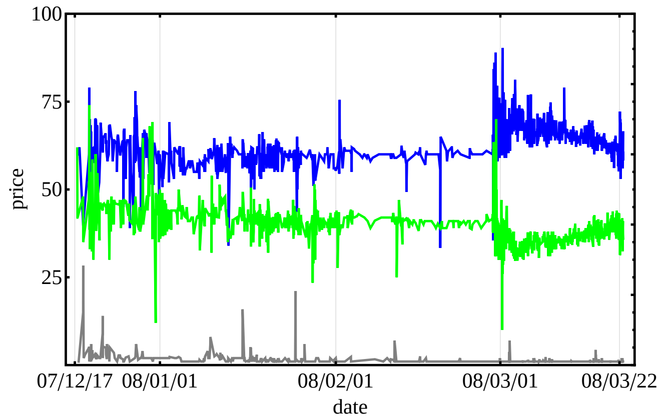

Fig. 1. The price time series during the whole experiment is shown. The contracts for candidates from KMT and DPP are drawn in blue and green, respectively, while the contract for invalid ballots is drawn in gray.

3 Results and analysis

In a previous study [23,24], we demonstrated that the transaction networks reconstructed from two parallel market experiments possess scale-free, hierarchical and disassortative structure. However, due to the low statistics in earlier experiments, we can not make solid conclusions for neither the dynamics of network growth nor the relation between individual wealth and network structure. Nevertheless, in the latest experiment, we have higher statistics and finer details about the trading information including the order submission, cancellation, expiration and transactions. We can therefore obtain the individual wealth or rebuild the transaction networks in many different ways. For example, the network in an arbitrary period of time or the sub-network consisting of only the trading of specific contract can be rebuilt without any difficulty.

3.1 Topology of transaction networks

Fig. 2 is a snapshot of the transaction network consisting of 24 nodes and 30 edges. The name on the node denotes the user name for each trader while the edges represent the transactions among them. From bunches of these snapshots, we notice that a hub (aggressive trader) develops quickly in our networks.

A network is classified as a scale-free network provided that its degree distribution function $P_d(k)$ decays as a power-law of the degree k. Namely, we have $P_d(k) \sim k^{-\gamma}$ where the exponent γ is a constant ranging from 1 to 3 for different kinds of networks [25]. However, due to the low statics in the empirical data, in alternative, we usually check the cumulative distributions defined as

$$\mathcal{P}_>(k) \equiv \sum_{k'=k}^{k_{max}} P_d(k'). \qquad (2)$$

Hence we have $\mathcal{P}_>(k) \sim k^{-(\gamma-1)}$ if $P_d(k) \sim k^{-\gamma}$. In Fig. 3, we plot $\mathcal{P}_>(k)$ for the transaction networks reconstructed

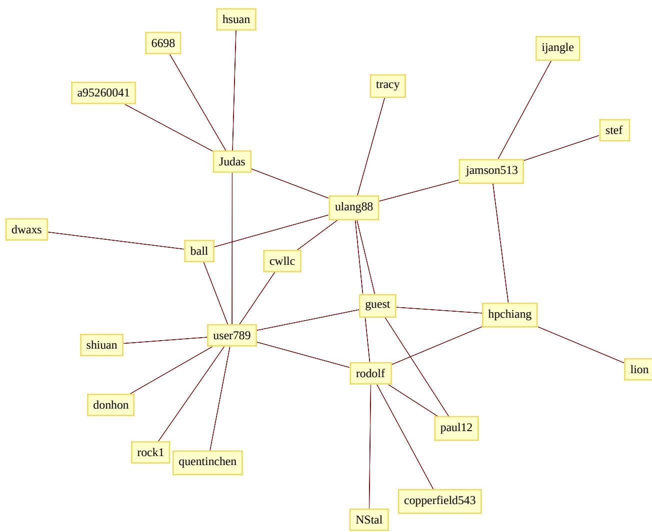

Fig. 2. A snapshot of the transaction network in our experiment. This network consists of 24 nodes and 30 edges.

in several ways. One should notice that we have normalized $\mathcal{P}_>(k)$ in order to make the comparison among these distributions. The largest network, shown as the black dot in Fig. 3(a), consisting of 1,985 nodes and 9,092 edges is the one that spans across the whole time horizon. The other three networks in Fig. 3(a) with the size $n = 426, 562$ and 588 are reconstructed from three non-overlapping periods, while the results of another three sub-networks for individual contracts are shown in Fig. 3(b). One can observed that all the distributions in Fig. 3 can be well fitted by a power-law decay with a corresponding exponent γ ranging from 2.16 to 2.31. Therefore, it is fair to assert that the transaction networks in our market experiment are scale-free and even we randomly extract a sub-network from the whole, the scale-free nature is still preserved.

In addition to the degree distribution, we also work out the average nearest neighbors degree (ANND) to see whether there exists the degree-degree correlation in our networks. The ANND is defined as

$$\langle k_{nn}(k)\rangle \equiv \sum_{k'=k}^{k_{max}} k' P_d(k'|k), \tag{3}$$

where $P_d(k'|k)$ denotes the conditional probability that an edge belonging to node with degree k links to another one with degree k' [26]. For uncorrelated networks, $\langle k_{nn}(k)\rangle = \langle k^2\rangle/\langle k\rangle$, independent of k. In Fig. 4, we plot the $\langle k_{nn}(k)\rangle$ for the transaction networks in 3 different periods of time. The $\langle k_{nn}(k)\rangle$ is shown to decay with increasing k, therefore we conclude that our networks are always disassortative, which means that the aggressive traders in our experiment tend not to trade with each other.

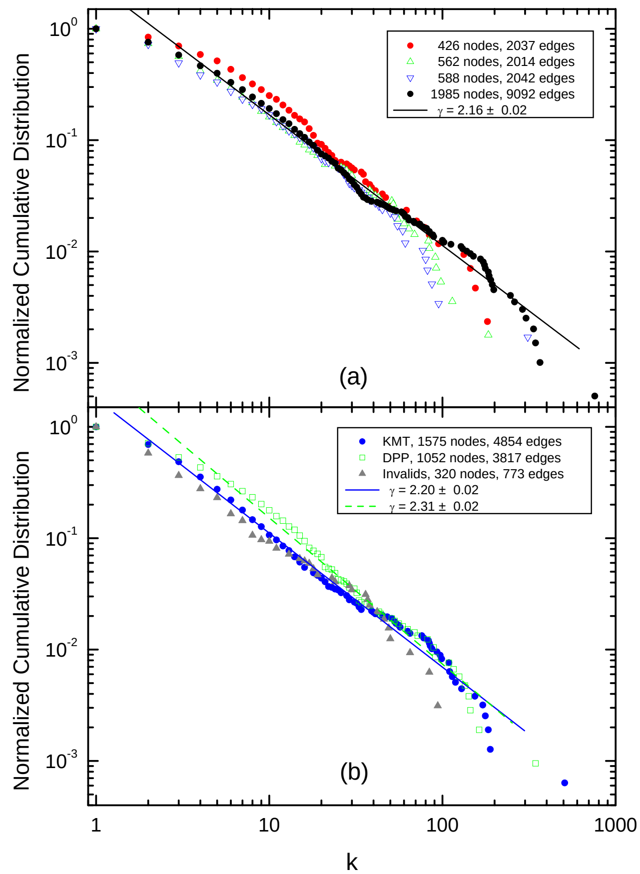

Fig. 3. The normalized cumulative degree distribution $\mathcal{P}_>(k)$ of our transaction networks are plotted where the solid lines denote the power-law fit. $\mathcal{P}_>(k)$ for four different periods are shown in (a) while the results for three individual contracts are plot in (b).

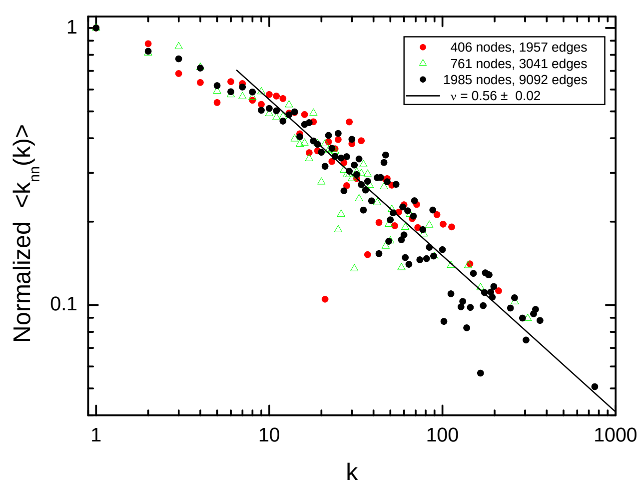

Fig. 4. The normalized $\langle k_{nn}(k)\rangle$ for our transaction networks in 3 different periods of time. The solid line is a fit of the form $\langle k_{nn}(k)\rangle \sim k^{-\nu}$ with $\nu = 0.52 \pm 0.02$.

3.2 Correlation between degrees and wealth

The wealth distribution functions $P_w(w)$ for both the traders with positive (black dot) and negative net incomes (red

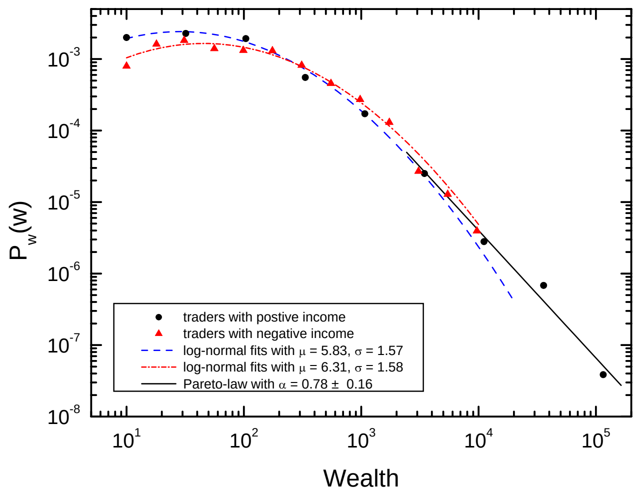

Fig. 5. The wealth distribution functions $P_w(w)$ for both the traders with postive (black dot) and negative net incomes (red triangle) are shown. The solid line denotes the Pareto-law tail for the rich with $\alpha = 0.78 \pm 0.16$ while the bulk is well fitted by a log-normal distribution.

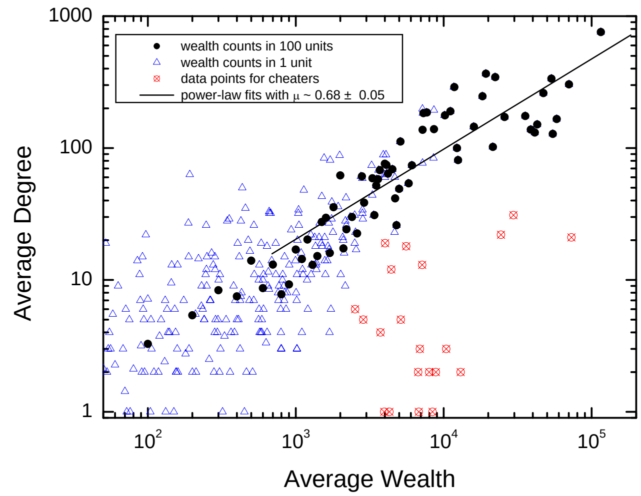

Fig. 6. The relation between the degree and wealth is shown. The black dot (blue triangle) denotes the wealth counted in units of 100 (1) while the red circle with cross indicates the cheaters in our experiment. The solid line is the power-law fit for the wealth beyond 2000 units which follows the form $k \sim w^{-\mu}$ with $\mu = 0.68 \pm 0.05$.

triangle) are shown in Fig. 5 where the richest guy accumulate 115,353 units of the virtual money in this experiment. One can observe that the distribution for the rich follows a power-law decay with a Pareto index $\alpha = 0.78 \pm 0.16$ while the remaining bulk can be well fitted by a log-normal distribution.

To further study the relationships between wealth and networks, we calculate the correlation between the degrees and wealth. The results are shown in Fig. 6 where the black dot (blue triangle) denotes the points for individual wealth counted in units of 100 (1) while the red circle with cross indicates the cheaters in our experiment. The suspect cheaters are first distinguished from the large deviation to the normal trend and then identified after checking their trading records. It is shown that the degrees k and wealth w are not strongly correlated until they reach above their critical values. We can thus obtain a power-law fit for the data with wealth beyond 2000 units which reads as $k \sim w^{-\mu}$ with $\mu = 0.61 \pm 0.06$. Combining all the findings and fitting parameters so far, we have

$$\begin{aligned} P_k(k) &\sim k^{-2.16\pm0.02}, \\ P_w(w) &\sim w^{-1.78\pm0.16} \quad \text{for } w > 2000, \\ k &\sim w^{0.68\pm0.05} \quad \text{for } w > 2000, \end{aligned} \tag{4}$$

and these three exponents can be related if we insert the third equation into the first one. The value $(0.68 \pm 0.05) \times (-2.16 \pm 0.02)$ falls between -1.35 and -1.59 which is roughly comparable with the exponent -1.78 ± 0.16 for the wealth distribution. Although the relations are not exact, we nevertheless shed the light on the possible explanation why Pareto's law persists across economies.

4 Conclusion

From a market experiment with human agents who can make decisions of their own free will, we explore the topology of transaction networks and demonstrate that it is scale-free and disassortative even for those with the size of the order of few hundred. It is also observed that the wealth distribution in our market follows the Pareto's law with a Pareto index $\alpha = 0.78 \pm 0.16$ for the rich a log-normal distribution for the remaining bulk. By further calculating the correlation between degrees and wealth, we argue that the exponent for the individual wealth, the degrees distribution and the wealth-degree correlation can be roughly related in our experiment. This finding may also be ture for the cases in the real world. Apart from that, we have also noticed that by probing the wealth-degree correlation one can readily identify the suspects of cheaters or the participants who benefit from the insider trading in our market, which might be useful against economic crimes in real financial markets. With more and more studies and efforts, we believe that the dynamics and interplay between transaction networks and wealth distribution may be revealed by our market experiments in the near future.

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